Integrated system reliability analysis

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<th>Title : Integrated system reliability analysis</th>
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<td><strong>Participants:</strong> AAU</td>
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**Relevant Description (3 lines):** Development of methodology for reliability and risk-based assessment of the wind turbine at system level integrating mechanical, electrical and structural components for innovative wind turbine systems.

**Specific targets:**
1) The report shall describe the state of the art of reliability and risk-based assessment of wind turbine components.
2) Development of methodology for reliability and risk-based assessment of the wind turbine at system level.
3) Describe quantitative and qualitative measures (indicators) that can be used to assess the reliability of innovations and new technologies.

**Measure of success:**
The methodology will be used as part of the assessment of the innovative designs which are developed in WP 2-4. These results will be reported in Deliverable 1.24. Success will be measured by the completeness of the methodology in addressing the specific targets of the deliverable and its sensitivity and appropriateness in expressing reliability-based indicators of the new designs.

**Participant Actions**
AAU: Collect relevant information, develop the methodology and report it.

**SMART GOALS**

S - specific

M - measurable, meaningful,

A - agreed upon, attainable, action-oriented

R - realistic, relevant, results-oriented

T - time-based, tangible, trackable
1. INTRODUCTION

Decisions related design and operation and maintenance of engineering systems such as wind energy structures can generally be performed at three levels of complexity:

- Risk-based: all benefits and costs during the whole life time are taken into account and the optimal decision is the one which maximizes the total expected benefits minus costs – or equivalently minimizes the total risks. The risks are obtained basically as a product of consequences (e.g. in Euros) and probabilities of relevant scenarios (events) incl. failure of components / the whole wind turbine. Risk-based decision making is also denoted a level IV method.

- Reliability-based: a decision problem is formulated where the cost (or weight) is minimized with a constraint on the reliability of the considered component(s). The reliability constraint is generally selected / calibrated on the basis of risk considerations and/or life safety requirements (applying level IV methods). The reliability can be estimated using e.g. structural reliability methods. Reliability-based decision making is also denoted level II and III methods, depending on the accuracy of the reliability method applied.

- Semi-probabilistic (deterministic): a decision problem is formulated where the cost (or weight) is minimized with constraints related to design load effects being smaller than design resistances. The design load effects and resistances are determined using safety factors (partial safety factors). The safety factors are generally selected / calibrated on the basis of reliability analysis (applying level II or II methods). Decision making on basis of semi-probabilistic (deterministic) techniques is also denoted level 1 methods.

It is noted that the (ISO, 1998) standard ISO 2394: ‘General principles on reliability for structures’ is currently being revised and the new, updated standard will be based on the principles described above.

The decision making can be applied at different stages and at different (sub) systems in the life time of a wind turbine, e.g.:

- At the design stage:
  - design of components minimizing the manufacturing costs
  - design of the whole wind turbine minimizing the manufacturing costs and maximizing the production of energy
  - design of the whole wind turbine minimizing the manufacturing, installation and operation & maintenance costs and maximizing the production of energy (minimize Cost Of Energy)
  - using design values determined based on international standards (e.g. the IEC 61400 standards)
  - design of the whole wind turbine minimizing the manufacturing, installation and operation & maintenance costs and maximizing the production of energy (minimize Cost Of Energy)
• using probabilistic / reliability-based methods accounting for uncertainties
• For an existing wind turbine:
  • Planning of operation and maintenance activities taking into account information from e.g. condition monitoring and inspections.

1.1. Scope and application

This document is produced within the frame of activities performed for WP.1 deliverable D1.34 of INNWIND.EU project.

This document defines a set of reliability analysis methods for reliability assessment of innovative wind turbine components and serves as a guideline to be used later-on within INNWIND.EU project. First, the indicators of reliability that could be used are described briefly in section 2, with more detailed descriptions in the following sections. Also, a procedure for reliability and risk-based qualification of innovations is presented.

As an example of application of the methodologies described in the report, two examples are given in sections 7 and 8. Section 7 describes the assessment of reliability of Multi Rotor System lattice support structure. Section 8 describes Operation and Maintenance aspects of the Multi Rotors System and the effect of different levels of individual rotor reliability on the overall availability of the Multi Rotor System.

1.1. Procedure for reliability and risk-based qualification of innovations

A recommended procedure for reliability- and risk-based qualification of innovations is presented in the report. It can be summarized as follows:

1. Apply the technology qualification procedure in section 5 together with methods in section 6 for
   a. Identification of components
   b. Identification and modeling of system of components
   c. Model uncertainties (physical, model, statistical and measurement) based on experience and tests dependent stage of development (based on the V-model)
   d. Model consequences of failure (in Euro)
2. Introduce simplifications (if needed) – such that only information expected to be relevant for assessment of innovative components are considered in the models
3. Estimate probability of failure of component
   a. Electrical and mechanical components: use failure rates from databases and/or tests, see section 3.1
   b. Structural components: use structural reliability methods (with simplified limit state equations and stochastic models, e.g. linearized limit state equations and Normal distributed stochastic variables), see section 3.2
4. Estimate probability of failure of system using methods in section 3.3
   a. Perform sensitivity analyses with focus on
      i. How important are innovative components for system reliability?
      ii. How important are uncertainties of components for system reliability?

5. Risk assessment, see section 4:
   a. Combination of failure probabilities with consequences (in Euro)
   b. Sensitivity analyses
   c. Relative comparison / ranking of (alternative) innovations.

It is noted that for decision making, it can be relevant to include existing components in the assessment such that a comparison between the innovation and the existing component / system can be made.
2. MEASURES (INDICATORS) OF RELIABILITY OF INNOVATIONS
   AND NEW TECHNOLOGIES

In the following section, the measures of reliability are discussed. Also, some
general measures of quality and acceptability are given. These measures can be
used to compare different designs/innovations of components and select the
most acceptable one.

2.1. Reliability index for structural components

The Reliability index $\beta$ (or probability of failure) can be used as one of the ways
of assessing the acceptability of structural components designs. Furthermore,
there are recommendations for reliability index and probability of failure in
industry standards (ISO, 1998) and (JCSS, 2002). Tables below show the
recommended target reliabilities for structural components:

**Table 2.1. Target reliability indices $\beta$ (annual rates) in accordance with (JCSS, 2002)**

<table>
<thead>
<tr>
<th>Relative costs of Safety measures</th>
<th>Minor consequences of failure</th>
<th>Moderate consequences of failure</th>
<th>Large consequences of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>$\beta = 3.1 \ (P_f \approx 10^{-3})$</td>
<td>$\beta = 3.3 \ (P_f \approx 5 \cdot 10^{-4})$</td>
<td>$\beta = 3.7 \ (P_f \approx 10^{-4})$</td>
</tr>
<tr>
<td>Normal</td>
<td>$\beta = 3.7 \ (P_f \approx 10^{-4})$</td>
<td>$\beta = 4.2 \ (P_f \approx 10^{-5})$</td>
<td>$\beta = 4.4 \ (P_f \approx 5 \cdot 10^{-6})$</td>
</tr>
<tr>
<td>Small</td>
<td>$\beta = 4.2 \ (P_f \approx 10^{-5})$</td>
<td>$\beta = 4.4 \ (P_f \approx 5 \cdot 10^{-6})$</td>
<td>$\beta = 4.7 \ (P_f \approx 10^{-6})$</td>
</tr>
</tbody>
</table>

**Table 2.2. Target reliability indices $\beta$ (annual rates) in accordance with (ISO, 1998)**

<table>
<thead>
<tr>
<th>Relative costs of Safety Measures</th>
<th>Consequences of failure</th>
<th>Moderate</th>
<th>Great</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\beta = 0$</td>
<td>$\beta = 1.5$</td>
<td>$\beta = 2.3$</td>
</tr>
<tr>
<td>Moderate</td>
<td>$\beta = 1.3$</td>
<td>$\beta = 2.3$</td>
<td>$\beta = 3.1$</td>
</tr>
<tr>
<td>Low</td>
<td>$\beta = 2.3$</td>
<td>$\beta = 3.1$</td>
<td>$\beta = 3.8$</td>
</tr>
</tbody>
</table>

Guidance on calculation of reliability indexes and probabilities of failure for
wind turbine components are given in section 3.2.

2.2. Reliability measures for electrical and mechanical components

The most important reliability measures for mechanical and electrical
components are as follows:

- The reliability (survivor) function, $R(t)$;
- The failure rate (hazard) function, $h(t)$;
- The Mean Time To Failure (MTTF);
A more detailed description on the calculation of these indicators for repairable and non-repairable components is given in section 3.1.

2.3. Expected lifetime, deterioration and respective uncertainties

The expected life, expressed in terms of Mean Time To Failure (MTTF) or in years of total Lifetime can be used as a measure of the Reliability of wind turbine components. Furthermore, the uncertainty of the expected life should be taken into account in order to adequately compare different designs. Figure 2.1 shows the uncertainty on expected life of two different designs. It is clearly visible that even though the second design yields a longer expected lifetime, it has higher uncertainty (\(\sigma_{\text{MTTF}}\)), and thus the actual lifetime is less predictable.

![Figure 2.1. Uncertainty of Expected Life.](image)

The Deterioration rate can also be used as a measure of design acceptance. Different degradations of components imply different required types of maintenance during the lifetime. E.g. components that are prone to linear degradation could be inspected less frequently throughout lifetime than components subject to exponential degradation. Further, components that tend to degrade exponentially, especially if there is an initiation period, should be subjected to preventive maintenance with shorter time intervals close to component expected lifetime. Figure 2.2 shows different degradation types.
2.4. Availability

Availability is a characteristic of a unit expressed by the probability that the unit will perform its required function under given conditions at a stated time. When wind turbines/farms are concerned, availability is of key importance, because it directly relates to produced power and therefore to the costs of energy. Furthermore, availability is largely dependent on the inherent reliability of wind turbine components. The following table shows different types of Availability with some comments, as stated in (Barbati, 2009).

Table 2.3. Quantitative measures of availability

<table>
<thead>
<tr>
<th>Measure</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
</table>
| Inherent Availability| $A = \frac{MTBF}{MTBF + MTTR}$ | - Where MTBF is the mean time between failure and MTTR is the mean time to repair  
- A probabilistic measure.  
- Reflects the percent of time a product would be available if no delays due to maintenance, supply, etc. (i.e., not design related) were encountered |
| Achieved Availability| $A = \frac{MTBM}{MTBM + MTTR_A}$ | - Where MTBM is the mean time between maintenance (preventive and corrective) and MTTR$_A$ is the mean time to accomplish preventive and corrective maintenance tasks  
- A probabilistic measure  
- Similar to A except that preventive and corrective maintenance are included |
Operational Availability

\[ A = \frac{MTBM}{MTBM + MDT} \]

- Where MTBM is the mean time between maintenance (preventive and corrective) and MDT is the mean downtime, which includes MTTR and all other time involved with downtime, such as delays.
- A probabilistic measure.
- Similar to inherent availability but includes the effects of maintenance delays and other non-design factors.
- \( A_o \) reflects the totality of the inherent design of the product, the availability of maintenance personnel and spares, maintenance policy and concepts, and other non-design factors, whereas \( A_i \) reflects only the inherent design.

Uptime ratio

\[ A = \frac{Uptime}{Uptime + Downtime} \]

- Uptime is the time that the product is in the customer’s possession and works; downtime is the total number of hours that the product is not operable/usable.
- A deterministic measure.
- Uptime Ratio is time-dependent the time period over which the measurement is made must be known.

2.5. Maintainability

Maintainability is a characteristic of an unit, expressed by the probability that preventive maintenance or repair of the item will be performed within a stated time interval for given procedures and resources (number and skill level of personnel, spare parts, test facilities, etc.). From a qualitative point of view, maintainability can be defined as the ability of an item to be retained in or restored to a specified state. The expected value (mean) of the repair time is denoted by MTTR (mean time to repair), and that of a preventive maintenance by MTTPM (mean time to preventive maintenance).

Reducing the MTTR is essential to achieve a system with higher maintainability. This can be achieved in many ways, some are listed below (Birolini, 2013):

1. Partition the equipment or system into line replaceable units (LRUs), often PCBs for electronic systems, and apply techniques of modular construction, starting from the functional structure; make modules functionally independent and electrically as well as mechanically separable; develop easily identifiable and replaceable LRUs which can be tested with commonly available test equipment.
2. Plan and implement a concept for automatic faults (failures and defects) detection and automatic or semiautomatic faults localization (isolation and diagnosis) down to the line replaceable unit (LRU) level, including hidden faults (failures & defects) and software defects as far as possible.
3. Aim for the greatest possible standardization of parts, tools, and testing equipment; keep the need for external testing facilities to a minimum.
4. Consider environmental conditions (thermal, climatic, mechanical) in field operation as well as during transportation and storage.
5. Plan and realize an appropriate logistic support including user documentation, training of operating & maintenance personnel, and logistic support in field.
6. Use quick fastening and unfastening mechanisms for service items.
7. Use common hand tools and a minimum number of hand tools for disassembly and reassembly.
8. Minimize serviceable items by placing the most likely items to fail, wear-out or need replacement in a small number of modules or assemblies. Design so that they require simple procedures to replace.
9. Use built-in self-test and indicators to quickly isolate faults and problems.
10. Eliminate or reduce the need for adjustment.
11. Use common, standard replacement parts.
12. Conceive operation and maintenance procedures to be as simple as possible, also taking into account personnel safety, describe them in appropriate manuals.
13. Provide self-latching access flaps of sufficient size; avoid the need for special tools (one-way screws, Allen screws, etc.); use clamp fastening.
14. Plan accessibility by considering the frequency of maintenance tasks.
16. Prevent faulty installation or connection through mechanical keying.
17. Use high standardization in selecting operational tools and make any labelling simple and clear.
18. Consider human aspects in the layout of operating consoles and in defining operating and maintenance procedures.
19. Order all steps of a procedure in a logical sequence and document these steps by a visual feedback.
20. Describe system status, detected fault, or action to be accomplished concisely in full text.
21. Avoid any form of hardware adjustment (or alignment) in the field, if necessary, carefully describe the relevant procedure.

2.6.Risk

Risk can be used as a measure of wind turbine component design acceptability, given that information about probabilities of failures and costs of failure consequences is available. A life cycle model should be used when possible. A detailed description on risk assessment and lifecycle modeling is given in section 4.
3. RELIABILITY ASSESSMENT OF WIND TURBINE COMPONENTS

Reliability of structural systems can be defined as the probability that the structure under consideration has a proper performance throughout its lifetime. Reliability methods are used to estimate the probability of failure. The information of the models which the reliability analyses are based on is generally not complete. Therefore the estimated reliability should be considered as a nominal measure of the reliability and not as an absolute number. However, if the reliability is estimated for a number of structures using the same level of information and the same mathematical models, then useful comparisons can be made on the reliability level of these structures. Further design of new structures can be performed by probabilistic methods if similar models and information are used as for existing structures which are known to perform satisfactory. If probabilistic methods are used to design structures where no similar existing structures are known then the designer has to be very careful and verify the models used as much as possible.

Generally the main steps in a reliability analysis are:

1. Select a target reliability level.
2. Identify the significant failure modes of the structure.
3. Decompose the failure modes in series systems of parallel systems of single components (only needed if the failure modes consist of more than one component). This means that each failure mode is decomposed in a sequence of components that each has to fail in the considered failure mode and these components therefore can be modeled as elements in a parallel system. Each failure mode / parallel system (sequence of failing components) can next be considered as an element in a series system. If the order of elements failing in the failure mode is changed then this results in a new parallel system in the series system.
4. Formulate failure functions (limit state functions) corresponding to each component in the failure modes.
5. Identify the stochastic variables and the deterministic parameters in the failure functions. Further specify the distribution types and statistical parameters for the stochastic variables and the dependencies between them.
6. Estimate the reliability of each failure mode.
7. In a design process change the design if the reliabilities do not meet the target reliabilities.
8. In a reliability analysis the reliability is compared with the target reliability.
9. Evaluate the reliability result by performing sensitivity analyses.

The reliability estimated as a measure of the safety of a structure can be used in a decision (e.g. design) process. A lower level of the reliability can be used as a constraint in an optimal design problem. The lower level of the reliability can be obtained by analysing similar structures designed after current design practice or it can be determined as the reliability level giving the largest utility (benefits –
costs) when solving a decision problem where all possible costs and benefits in the expected lifetime of the structure are taken into account.

In order to be able to estimate the reliability using probabilistic concepts it is necessary to introduce stochastic variables and/or stochastic processes/fields and to introduce failure and non-failure behaviour of the structure under consideration. The uncertainty modelled by stochastic variables/fields can be divided in the following groups:

**Physical uncertainty:** or inherent uncertainty is related to the natural randomness of a quantity, for example the uncertainty in the yield stress due to production variability.

**Measurement uncertainty:** is the uncertainty caused by imperfect measurements of for example a geometrical quantity.

**Statistical uncertainty:** is due to limited sample sizes of observed quantities.

**Model uncertainty:** is the uncertainty related to imperfect knowledge or idealizations of the mathematical models used or uncertainty related to the choice of probability distribution types for the stochastic variables.

### 3.1. Reliability assessment of electrical and mechanical components

Generally when electrical components of wind turbines are concerned, there is little or no information available about the exact failure mechanisms that govern the failures, the mechanisms are very uncertain or too complicated to be expressed in terms of failure functions. Furthermore, most electrical, electronic and mechanical components deteriorate during their lifetime due to elevated operating temperatures, chemical changes, mechanical wear, fatigue, overloading etc. Failure of particular component can occur for any one of these reasons, a combination of different reasons and even may be caused indirectly by deterioration of some other component (Thoft-Christensen & Baker, 1982). Therefore, classical reliability theory should be used when assessing the reliability level/probability of failure of electrical and mechanical wind turbine components and assemblies. (Lamberson, 2003). This is usually done by using existing knowledge about the reliability of similar components (same technology used in different application/environment, previous generation of the technology in question). Information regarding these components is available in failure databases or handbooks and is expressed in terms of failure rates or Mean Time To Failure (MTTF). Below a listing of useful failure sources is given:

- **OREDA** – database containing information for components and systems used in offshore oil and gas industry (SINTEF, 2009).
- **WMEP** – database containing information form 1500 wind turbines (Germany), (Bard, et al., 2011).
- LWK - database containing information from 630 wind turbines (Germany), (Pettersso, et al., 2010).
- WINDSTATS – data from ~7000 wind turbines from Germany and Denmark (DOWEC, 2003).
- VTT – failure statistics from Finland. (Stenberg, 2010)
- Vindstat – database from >700 wind turbines in Sweden (Pettersso, et al., 2010).

Figure 3.1. Mechanical failure rate distribution according to WindStats, adopted from (DOWEC, 2003).

Where no information about the component in question is available, tests should be conducted in order to obtain required time to failure (failure rate) statistics.

**Introduction to Classical Reliability Analysis**

Classical reliability theory was developed to estimate statistical characteristics of the life of technical components for design and operation of systems of such components. The characteristics are expected failure rate, expected life, the Mean Time To Failure (MTTF), probability of failure, etc. The probability of failure of a component can be expressed in terms of the reliability function \( R(t) \):

\[
R(t) = 1 - F_T(t) = 1 - P(T \leq t) \quad (3.1)
\]

\[
R(t) = 1 - \int_{0}^{t} f_T(\tau) d\tau = \int_{t}^{\infty} f_T(\tau) d\tau \quad (3.2)
\]

where \( T \) is a random variable describing time to failure and \( F_T(t) \), \( f_T(t) \) are its cumulative probability density and probability density functions.

It is seen that \( R(t) \) depends on the type of distribution used to describe time to failure. When the failure rate can be assumed to be independent of time (constant) exponential distribution can be used:
When the failure rate is increasing (deterioration of components) or decreasing (burn-in, “infant mortality”) in time, Weibull distribution can be used to model time to failure:

\[
R(t) = 1 - F_T(t) = 1 - \left(1 - e^{-\lambda t}\right) = e^{-\lambda t}
\]  
(3.3)

The parameters of these distributions should be estimated on the basis of observed time to failure of components in question e.g. by Maximum-Likelihood methods or used directly from failure databases given that the type of distribution is provided also. When the distribution of the time to failure is defined, expected life of the component can be derived:

\[
E[T] = \int_0^T f_T(\tau) \, d\tau = \int_t^\infty R(t) \, dt
\]
(3.5)

The expected life is usually referred to as Mean Time To Failure (MTTF) for non-repairable components. When the components are repairable Mean Time Between Failures can be introduced (MTTR is Mean Time To Repair):

\[
MTBF=MTTF+MTTR
\]
(3.6)

The failure rate is a measure of how the probability of failure changes as a function of time. By using the hazard function \(h(t)\), defined as an instantaneous failure rate, typical behavior of majority of technical components can be modelled:

\[
h(t) = \lim_{\delta t \to 0} \frac{R_T(t) - R_T(t + \delta t)}{\delta t R_T(t)} = \frac{f_T(t)}{R(t)}
\]
(3.7)

For many wind turbine components subject to degradation / damage accumulation the bath-tub model in Figure 3.2 can be used to illustrate the development of the failure rate during the lifetime. Initially a high failure rate can be expected due to fabrication / burn-in defects. Next, a period with a ‘normal’ constant failure / defect rate will take place. At the end of the lifetime of the component the failure / defect rate can be expected to increase.
It should be mentioned that when the failure rate $\lambda$ is constant, MTTF can be defined as follows:

$$MTTF = \frac{1}{\lambda} \quad (3.8)$$

**Effects of different environment and operational conditions on Reliability**

Since information given in different databases and handbooks is in general not directly applicable because of different environmental or operational conditions of the component in question, the MTTF’s and failure rates have to be adjusted. As an example for electrical components, this can be done by recalculating the failure rate (Department Of Defence, USA, 1990):

$$\lambda_{\text{actual}} = \lambda_{\text{base}} \pi_{E,Q,R...} \quad (3.9)$$

where $\lambda_{\text{actual}}$ and $\lambda_{\text{base}}$ are adjusted and base failure rates, and $\pi_{E,Q,R...}$ are different influence factors, describing how the failure rate changes with changing environment, quality, operational conditions, etc. More equations for adjusted failure rate of electrical components can also be found in ReliaWind report on Reliability analysis methods (Barbati, 2009).

Similar approach can be used for mechanical components, more information can be found in (Carberoc Division, 2010).

**3.2. Reliability assessment of structural components**

In general, the reliability analysis of structural components is fundamentally different from mechanical/electrical components. This is due to the fact that failures of structural components (structural failures) are very rare events and tend to occur as a consequence of and extreme event (extreme loading) or deterioration of the structure (e.g. fatigue or corrosion). Therefore the
information about the reliability of these components cannot be obtained from databases and Structural Reliability Theory has to be used in order to quantify the reliability level.

Typical failure modes to be considered in a reliability analysis of a structural system are yielding, buckling (local and global), fatigue and excessive deformations. The failure modes (limit states) are generally divided in:

**Ultimate limit states**: Ultimate limit states correspond to the maximum load carrying capacity which can be related to e.g. formation of a mechanism in the structure, excessive plasticity, rupture due to fatigue and instability (buckling).

**Conditional limit states**: Conditional limit states correspond to the load-carrying capacity if a local part of the structure has failed. A local failure can be caused by an accidental action or by fire. The conditional limit states can be related to e.g. formation of a mechanism in the structure, exceedance of the material strength or instability (buckling).

**Serviceability limit states**: Serviceability limit states are related to normal use of the structure, e.g. excessive deflections, local damage and excessive vibrations.

**Introduction to Structural Reliability Analysis**

Generally, methods to measure the reliability of a structure can be divided in four groups, see (Madsen, et al., 1986):

- **Level I methods**: The uncertain parameters are modeled by one characteristic value, as for example in codes based on the partial safety factor concept.
- **Level II methods**: The uncertain parameters are modeled by the mean values and the standard deviations, and by the correlation coefficients between the stochastic variables. The stochastic variables are implicitly assumed to be normally distributed. The reliability index method is an example of a level II method.
- **Level III methods**: The uncertain quantities are modeled by their joint distribution functions. The probability of failure is estimated as a measure of the reliability.
- **Level IV methods**: In these methods the consequences (cost) of failure are also taken into account and the risk (consequence multiplied by the probability of failure) is used as a measure of the reliability. In this way different designs can be compared on an economic basis taking into account uncertainty, costs and benefits.

Level I methods can e.g. be calibrated using level II methods, level II methods can be calibrated using level III methods, etc. Level II and III reliability methods are considered in these notes. Several techniques can be used to estimate the reliability for level II and III methods, e.g.
• **Simulation techniques:** Samples of the stochastic variables are generated and the relative number of samples corresponding to failure is used to estimate the probability of failure. The simulation techniques are different in the way the samples are generated.

• **FORM techniques:** In First Order Reliability Methods the limit state function (failure function) is linearized and the reliability is estimated using level II or III methods.

• **SORM techniques:** In Second Order Reliability Methods a quadratic approximation to the failure function is determined and the probability of failure for the quadratic failure surface is estimated.

Within the framework of Structural Reliability, information about (as much as possible) all external and internal influences acting on the structure is required in order to perform a reliability assessment. It is therefore necessary to establish probabilistic models for all these quantities. The fundamental quantities that characterize the behavior of a structure are called the basic variables and are denoted \( X = (X_1, \ldots, X_n) \) where \( n \) is the number of basic stochastic variables. Typical examples of basic variables are loads, strengths, dimensions and model uncertainties. The basic variables can be dependent or independent. A stochastic process can be defined as a random function of time such that for any given point in time the value of the stochastic process is a random variable. Stochastic fields are defined in a similar way where the time is exchanged with the space.

The joint density function for the stochastic variables \( X \) is denoted \( f_X(x) \). The elements in the vector of expected values and the covariance vector are:

\[
\mu_i = E[X_i] \quad , \quad i = 1, \ldots, n \tag{3.10}
\]

\[
C_{ij} = \text{Cov}[X_i, X_j] \quad , \quad i, j = 1, \ldots, n \tag{3.11}
\]

The standard deviation of \( X_i \) is denoted \( \sigma_i \). The variance of \( X_i \) is \( \sigma_i^2 = C_{ii} \). The coefficient of correlation between \( X_i \) and \( X_j \) is defined by:

\[
\rho_{ij} = \frac{C_{ij}}{\sigma_i \sigma_j} \quad , \quad i, j = 1, \ldots, n \tag{3.12}
\]

with \(-1 \leq \rho_{ij} \leq 1\).

Application of FORM, SORM and simulation methods requires as noted above that it is possible for given realizations \( x \) of the basic variables to state whether the structure (or component/failure mode) is in a safe state or in a failure state. The basic variable space is thus divided into two sets, the safe set \( \omega_S \) and the
failure set $\omega_F$. The two sets are separated by the failure surface (limit state surface). It is assumed that the failure surface can be described by the equation:

$$g(x) = g(x_1, \ldots, x_n) = 0$$

where $g(x)$ is denoted the failure function.

Usually the failure function is defined such that positive values of $g$ correspond to safe states and negative values correspond to failure states, see Figure 3.3.

$$g(x) \begin{cases} > 0 & , \; x \in \omega_s \\ \leq 0 & , \; x \in \omega_f \end{cases}$$

(3.13)

![Figure 3.3. Failure function $g(x)$.](image)

It is important to note that the failure surface does not define a unique failure function, i.e. the failure surface can be described by a number of equivalent failure functions. However, whenever possible, differentiable failure functions should be used. In structural reliability the failure function usually results from a mechanical analysis of the structure.

If, in the failure function $x$ is replaced by the stochastic variables $X$, the so-called safety margin $M$ is obtained:

$$M = g(X)$$

(3.14)

$M$ is a stochastic variable. The probability of failure $P_f$ of the component is:

$$P_f = P(M \leq 0) = P(g(X) \leq 0) = \int_{\omega_f} f_X(x)dx$$

(3.15)
Reliability Analysis for Linear Safety Margins

A safety margin, which is linear in basic variables, can be written:

\[ M = a_0 + a_1 X_1 + \cdots + a_n X_n \]  

(3.16)

where \( a_0, a_1, \ldots, a_n \) are constants. The expected value \( \mu_M \) and the standard deviation \( \sigma_M \) are:

\[ \mu_M = a_0 + a_1 \mu_{X_1} + \cdots + a_n \mu_{X_n} = a_0 + a^T \mu_X \]  

(3.17)

\[ \sigma_M = \sqrt{a^T Ca} \]  

(3.18)

If the basic variables are independent (3.18) simplifies to:

\[ \sigma_M = \sqrt{a_1^2 \sigma_{X_1}^2 + \cdots + a_n^2 \sigma_{X_n}^2} \]  

(3.19)

As a measure of the reliability of a component with the linear safety margin (3.10) the reliability index \( \beta \) can be used:

\[ \beta = \frac{\mu_M}{\sigma_M} \]  

(3.20)

This definition of the reliability index was used by (Cornell, 1966).

If the basic variables are normally distributed and the safety margin is linear then \( M \) becomes normally distributed. The probability of failure is, see figure 3.3:

\[ P_f = P(M \leq 0) = P(\mu_M + U \sigma_M \leq 0) = P \left( U \leq -\frac{\mu_M}{\sigma_M} \right) = \Phi(-\beta) \]  

(3.21)

where \( \Phi \) is the standard normal distribution function and \( U \) is a standard normally distributed variable with expected value zero and unit standard deviation (\( \mu_U = 0, \sigma_U = 1 \)).

Figure 3.4. Illustration of reliability index and probability of failure. \( \varphi \) is the standard normal density function.
When the safety margins are not linear or the basic variables are correlated, a detailed description on how to evaluate the reliability can be found in Appendix A.

More detailed information about structural reliability methods can be found in the following textbooks: (Madsen, et al., 1986), (Melchers, 1987), (Thoft-Christensen & Baker, 1982), (Ditlevsen & Madsen, 1996) and (Sørensen, 2011).

### 3.3. System Reliability assessment

Generally, when talking about the reliability level of a whole wind turbine, reliability of electrical/mechanical components has to be combined with reliability of structural components. On a global scale wind turbine can be modelled as a sequence of parallel systems of components connected in a series system (Figure 3.5).

![Figure 3.5. System reliability model as a series system of parallel systems.](image)

It is clear that if one of these parallel systems fails, whole system fails. Also, the constituents of the series system in Figure 3.5 can be regarded as different assemblies of a wind turbine (rotor, nacelle, tower, substructure assemblies etc.). Furthermore, every assembly of a wind turbine can also be represented as series system of parallel systems at component level. Therefore, every element in the global series system can be evaluated individually, based on the type of component using either classical or structural reliability.

Decomposing the global system (wind turbine) into layers of series and parallel systems allows for a simple representation and evaluation of the overall reliability by using simple rules of probability (given in section 3.1, some example are also provided in Figure 3.6). Guidance on wind turbine decomposition into assemblies and components can be found in sections 6.1, 6.2 and 6.3.
The approaches to be used in reliability assessment of systems differ for structural and mechanical/electrical components. In the following subsections the system reliability will be discussed separately for mechanical/electrical and structural components.

### 3.3.1. Mechanical and electrical components

When components of wind turbines are concerned, they can be considered to be repairable. When failures of the components can be considered statistically independent, the reliability of series and parallel systems can be estimated by:

For series systems:

\[
R_s(t) = \prod_{i=1}^{n} R_i(t) \quad \text{for series systems;} \quad (3.22)
\]

For parallel systems:

\[
R_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t)) = \prod_{i=1}^{n} R_i(t) \quad \text{for parallel systems;} \quad (3.23)
\]

Also, when redundancy is incorporated in the design, reliability of such a system can be modelled as k-out-of-n structure and calculated as follows (given that the reliabilities of the components within a redundant system are identical):

\[
R_s(t) = \sum_{y=1}^{n} \binom{n}{y} R_s(t)^y (1 - R_s(t))^{n-y} \quad (3.24)
\]

When the reliabilities of the components of the system cannot be considered independent (e.g. common cause failures) couple of methods for system reliability assessment are available:

1. The Square-Root method.
2. The Beta-Factor method.
3. The Binomial Failure Rate model (BFR).
More information on these models can be found in (Rausand & Hoyland, 2004).

It is noted that the assumption of statistical independence (or sometimes fully dependency) between components is not always fulfilled, and the above simplified models for estimating the system reliability cannot be used. Instead the approaches used for structural components can be applied implying more complicated calculations.

### 3.3.2. Structural components

Since reliability of structural components is estimated using their respective limit states, now the limit states can be combined and system reliability can be estimated. Below FORM-approximation of reliability is given. Modelling of series and parallel systems is explained in Appendix B.

**FORM Approximations of the System Reliability**

Considering series systems of $m$ components and parallel systems of $n$ components, the following equations can be used to assess the probability of failure (and further-on – the reliability in terms of reliability index $\beta$). The equations are based on application of FORM (First Order Reliability Methods).

For series systems:

$$
P_f^S = P\left(\bigcup_{i=1}^{m}\{M_i \leq 0\}\right) = P\left(\bigcup_{i=1}^{m}\{g_i(X) \leq 0\}\right) = P\left(\bigcup_{i=1}^{m}\{g_i(T(U)) \leq 0\}\right)
$$  \quad (3.25)

$$
P_f^S \approx P\left(\bigcup_{i=1}^{m}\{-\mathbf{a}_i^T\mathbf{U} \leq -\beta_i\}\right)
$$  \quad (3.26)

$$
\beta^S = -\Phi^{-1}(P_f^S) = -\Phi^{-1}(1 - \Phi_m(\beta; \rho))
$$  \quad (3.27)

where $\Phi_m$ is the $m$-dimensional normal distribution function.

For parallel systems:

$$
P_f^P = P\left(\bigcap_{i=1}^{n}\{M_i \leq 0\}\right) = P\left(\bigcap_{i=1}^{n}\{g_i(X) \leq 0\}\right) = P\left(\bigcap_{i=1}^{n}\{g_i(T(U)) \leq 0\}\right)
$$  \quad (3.28)

$$
P_f^P \approx P\left(\bigcap_{i=1}^{n}\{\mathbf{b}_i - \mathbf{a}_i^T\mathbf{U} \leq 0\}\right) = P\left(\bigcap_{i=1}^{n}\{-\mathbf{a}_i^T\mathbf{U} \leq -\beta_i\}\right) = \Phi_{n_d}(-\mathbf{\beta}^P; \rho)
$$  \quad (3.29)

$$
\beta^P = -\Phi^{-1}(P_f^P) = -\Phi^{-1}(\Phi_{n_d}(-\mathbf{\beta}^P; \rho))
$$  \quad (3.30)

where $\Phi_{n_d}$ is the $n_d$-dimensional ($n_d$ - number of active failure modes at the beta point) normal distribution function. It has been used that the correlation
coefficient $\rho_{ij}$ between two linearized safety margins $M_i = \beta_i - \alpha_i^T U$ and $M_j = \beta_j - \alpha_j^T U$ is $\rho_{ij} = \alpha_i^T \alpha_j$.

Considering a configuration as shown in Figure 3.5, the reliability of a series system of parallel systems can also be estimated, using the following equations:

$$P_f^S = P\left( \bigcap_{i=1}^{n_p} \bigcup_{j=1}^{m} \{ g_{ij}(X) \leq 0 \} \right)$$

$$\rho^S = -\Phi^{-1}(1 - \Phi_{n_p}(\beta^P; \rho^P))$$

where $\beta^P$ is an $n_p$-vector of generalized reliability indices for the individual parallel systems and $\rho^P$ is a matrix of the corresponding approximate correlation coefficients between the parallel systems.

The multi-dimensional integral in equations (3.25-3.32) can only in special cases be solved analytically and will for even small dimensions, say five, be too costly to evaluate by numerical integration. Instead, so-called bounds methods are used for hand calculations and so-called asymptotic approximate methods are used for computational calculations. The calculation of bounds for reliability of systems is presented in Appendix B.

**Comments on General Systems Reliability Models for structural components**

The reliability modelling of a general system as a series system of parallel systems is healthy seen from a reliability theoretical point of view but from a structural engineering point of view in many cases unrealistic. This is due to the fact that the parallel systems reliabilities are dependent on the history of the load effects in the individual elements or in other words on 1) the residual load carrying capacity of a failed element or elements and 2) how the overall load effects in the entire structure are redistributed at each step in a sequence of element failures. This leads to the conclusion that failure of more than one structural element of major importance often cannot be treated in a realistic manner. More generally it can be said that the systems reliability model is totally dependent of the structural response model and thus it should not be refined more than the structural response model justifies.
4. RISK ASSESSMENT AND LIFE CYCLE MODELLING

4.1. Risk assessment

In the following some basic concepts of risk analysis is described. A more detailed description can be found in the JCSS Guidance paper on risk analysis (JCSS, 2008). Engineering facilities including wind turbines are all intended to contribute to the benefit and quality of life. Therefore when such facilities are planned it is important that the benefit of the facility can be identified considering all phases of the life of the facility, i.e. including design, manufacturing, construction, operation and eventually decommissioning.

Risk is here defined as the expected consequences associated with a given activity. Considering an activity with only one event with potential consequences, risk $R$ is thus defined as the probability that this event will occur $P$ multiplied with the consequences given the event occurs $C$ i.e.

$$R = P \cdot C$$

(4.1)

For an activity with $n$ events the risk is defined by:

$$R = \sum_{i=1}^{n} P_i \cdot C_i$$

(4.2)

where $P_i$ and $C_i$ are the probability and consequence of event $i$.

This definition is consistent with the interpretation of risk used e.g. in the insurance industry (expected losses) and risk may e.g. be given in terms of Euros, dollars, number of human fatalities, etc.

![Figure 4.1. Principal flow diagram of risk assessment.](image)

Risk assessment is used in a number of situations with the general intention to indicate that important aspects of uncertainties, probabilities and / or frequencies and consequences have to be considered in some way or other. Decision theory provides a theoretical framework for such analyses, see Figure 4.1.
In typical decision problems encountered the information basis is often not very precise. In many situations it is necessary to use historical data. The available historical information is often not directly related to the problem considered but to a somewhat similar situation. Furthermore, an important part of a risk assessment is to evaluate the effect of additional information, risk reducing measures and/or changes of the considered problem. It is therefore necessary that the framework for the decision analysis can take these types of information into account and allow decisions to be updated based upon new information. This is possible if the framework of Bayesian decision theory is used, see e.g. (Raiffa & Schlaifer, 1968) and (Benjamin & Cornell, 1970).

A fundamental principle in decision theory is that optimal decisions must be identified as those resulting in the highest expected utility, see e.g. (Ditlevsen & Madsen, 1996). In typical engineering applications the utility may be related to consequences in terms of costs, fatalities, environmental impact etc. In these cases the optimal decisions are those resulting in the lowest expected costs, the lowest expected number of fatalities and so on.
Risk analyses can be presented in a format, which is almost independent from the application. Figure 4.2 shows a general scheme for risk analysis, see (Stewart & Melchers, 1997). One of the most important steps in the process of a risk analysis is to identify the context of the decision problem:

- Who are the decision maker(s) and the parties with interests in the activity (e.g. society, client(s), state and organizations)?
- Which matters might have a negative influence on the impact of the risk analysis and its results?
- What might influence the manner in which the risk analysis is performed (e.g. political, legal, social, financial and cultural)?

Furthermore, the important step of setting the acceptance criteria must be performed. This includes the specification of the accepted risks in regard to economic, public or personnel safety and environmental criteria. In setting the acceptable risks – which might be considered a decision problem itself, account should be taken to regulations in the considered application area.

4.2. Life cycle modelling

The Life-cycle approach can be used to describe the flow of income and costs with the following main steps:
- Project development
- Planning
- Investigations & tests
- Design
- Manufacturing
  - Wind turbine fabrication
    - Rotor
    - Nacelle
    - Gearbox
    - Generator
    - Power converter
  - Substructure
    - Tower
    - Foundation
  - Electrical connection/cables, …
- Installation
- Operation
  - Operation & Maintenance costs
  - Energy production (income)
- Decommissioning
It is noted that the lifecycle constituents can be grouped in different ways. When describing the constituents of the life-cycle it is important to define the overall system to be considered (e.g. single wind turbine / wind farm). Further it is important to identify the decision makers (stake holders) – e.g. WT manufacturer, developer, owner of WT, society. A general framework for formulation of the life-cycle approach can e.g. be found in JCSS (2008).

As mentioned in before different formulations as basis for optimal decision making can be used with increasing requirements for information:

a) crude deterministic (level I)  
b) deterministic, code/standard-based formulation (level I)  
c) reliability-based formulation (level II and III)  
d) risk-based formulation (level IV)

The design parameters can be defined at different levels of detail. The key design drivers could be selected as:

- Rotor diameter  
- Hub height  
- Tip speed  
- Wind turbine separation (in wind farm)

The main design parameters are denoted $z=(z_1, z_2, ..., z_n)$ and include parameters describing among others the wind turbine, the wind farm, the support structure and the operation & maintenance strategy.

In a more detailed modelling the following parameters could be added to the list of design parameters:

- cross-sectional dimensions defining geometry of blades, tower,…  
- O&M strategy

Some design conditions are often fixed for a given site, but could in some cases also be considered as design parameters:

- Magnitude of wind farm (in terms of MW and/or geographic area of wind farm)  
- Wind climate (incl. terrain): mean wind speed + turbulence  
- Wave and current climate (offshore)  
- Water depth  
- Soil conditions  
- Distance from land (or nearest harbor)

Further, some parameters in the decision problem will be subject to uncertainty, e.g. annual maximum wind speed and strength of steel. These uncertain parameters can be modelled by stochastic variables denoted $X=(X_1, X_2, ..., X_N)$. 

In deterministic (semi-probabilistic) design fixed values of the stochastic variables are used, denoted design values \( X_{d,i} = (X_{d,1}, X_{d,2}, \ldots, X_{d,n}) \).

In a cost-benefit analyses (or risk analyses) the total expected cost-benefits in its expected lifetime are to be maximized. This can be formulated by the following optimization problem:

\[
\max_{z} W(z) = B(z) - \left( C_I(z) + C_{OM}(z) + C_F(z) + C_{DEM}(z) \right)
\]  

(4.3)

where:
- \( z \) represents design/decision variables. Examples: cross-sectional dimensions of tower, time interval and type of service
- \( B \) expected capitalized (discounted) benefits (electricity production) during the lifetime:
  \[
  B = \sum_{t=1}^{T_L} B_t \frac{1}{(1+r)^t}
  \]  
  (4.4)
- \( B_t \) income/benefit from electricity produced in year \( t \)
- \( r \) rate of interest
- \( t \) time (in years)
- \( T_L \) expected lifetime, e.g. 20 years
- \( C_I \) initial costs
- \( C_{OM} \) expected capitalized OM costs, see also section 4:
  \[
  C_{OM} = \sum_{t=1}^{T_L} C_{OM,t} \frac{1}{(1+r)^t}
  \]  
  (4.5)
- \( C_{OM,t} \) costs to OM in year \( t \)
- \( C_F \) expected capitalized costs to collapse:
  \[
  C_F = \sum_{t=1}^{T_L} P_{F,t} C_F \frac{1}{(1+r)^t}
  \]  
  (4.6)
- \( C_F \) costs due to collapse
- \( P_{F,t} \) probability of collapse in year \( t \)
- \( C_{DEM} \) demolition costs: actual costs in year \( T_L \) multiplied by \( \frac{1}{(1+r)^{T_L}} \)

Note that expected values of the costs to OM and collapse and of the benefits are to be determined taking into account uncertainties, i.e. using probabilistic measures. OM could also include costs to inspections and repairs. The related decision parameters could include inspection times, qualities and locations.

More information on life cycle modelling can be found in Appendix C.
5. GENERAL FRAMEWORK FOR TECHNOLOGY QUALIFICATION

Technology Qualification is a process (TQP) of providing sufficient evidence that a technology will function within specified limits and with acceptable and manageable risks. Generally TQP has to be applied to all newly developed technologies in cases where there are no directly applicable standards or guidelines. Otherwise, if appropriate standards can be used, a technology is considered to be qualified if it complies with the requirements of those standards.

Since the purpose of this report is to give insight on how to assess reliability of newly developed or partially altered innovative wind turbine components, TQP can be used as the underlying methodology to provide proof of sufficient reliability of different wind turbine components. Following the TQ framework throughout the technology development cycle it is possible to quantify the change in reliability level of a given component, provided that proper tools are used for reliability assessment. Figure 5.1 represents the change in probability of failure and related uncertainty of arbitrary technology as qualification progresses.

![Figure 5.1. Change of probability of failure and related uncertainties. Adopted from (DNV, 2013).](image)

Throughout the development process novel technical solutions usually progress through several phases (concept evaluation, pre-engineering, etc.). Depending on the stage of technology development, multiple TQ processes can be defined subsequently in order to ensure a traceable and reliable stepwise development process. These multiple TQ processes should have goals defined according to a particular development phase. Milestones are reached after each development phase is completed and moving forward to the next phase is possible. An example of such TQ program is shown in Figure 5.2.
Furthermore, TQ process can be used irrespective of the scale of the technology. When qualification is needed for a component of a complex system, e.g. a generator for a wind turbine, a separate TQ program can be introduced and later on incorporated in a higher level TQP.

5.1. Technology Qualification requirements based on DNV-RP-A203

DNV-RP-A203 is one of the most largely used methodologies for qualification of new (innovative) technology. The last version of the guidelines was published in 2013 by Det Norkse Veritas and is considered to be the state of the art when it comes to new technology assessment. The guideline can be used for any kind of technology without limitations of nature or function of a given technology (i.e. electrical, mechanical, structural components of wind turbines, software solutions etc.). The generic results of TQP, as stated in (DNV, 2013) are:

- determination of the probability density distribution of the service lifetime of given technology;
- determination of reliability level;
- definition of operational margins against specific failure modes or specific performance targets.
The basic TQ process is shown in Figure 5.3, it has 6 distinct steps that have to be followed and thoroughly documented.

1. **Technology qualification basis**: identifying the technology, its function and intended use, expectations to the technology.
2. **Technology assessment**: categorizing the degree of novelty to focus the efforts where the related uncertainty is most significant, identify key challenges and uncertainties.
3. **Threat assessment**: identify the threats and failure modes together with their risks.
4. **Technology qualification plan**: developing a plan containing the qualification activities addressing the identified risks.
5. **Execution of the plan**: executing the activities specified in step 4. Evidence is collected from experience (expert knowledge), numerical analysis and physical tests.
6. **Performance assessment**: assess whether the evidence produced meets the requirements in step 4.

**Figure 5.3. Basic Technology Qualification Process, (DNV, 2013)**

Feedback loops within the process imply an iterative nature of TQP. The iterations are necessary when primary design is changed (Technology Modification) in order to improve safety, reliability, cost, operation etc. Iterations are also used when unanticipated failure modes are revealed while qualification progresses. Technology Modification will only be implemented if it has a defined purpose, to name a few:

- remove a failure mode;
- reduce the probability or consequences of failure;
- reduce the total cost;
- improve confidence etc.
Technology Composition analysis

Since wind turbine as a whole is a very complex system, it is very important to break it down to manageable component systems or even to individual sub-components. This process is referred to as Technology Composition analysis in (DNV, 2013). This step is very important also because not all of wind turbine components fall under the definition of “new technology”. System decomposition can be performed taking into account:

- functions and sub-functions (without reference to technical solutions used to deliver the function);
- systems, sub-systems and components with functions;
- process sequences and operations.

For highly complex systems, a system engineering approach is recommended using a hierarchical structure linking the technology expectations with functions and sub-functions. System analysis using hardware components or units should be performed. The software used is also analyzed separately.

TQP can be implemented for the whole wind turbine or separately for different assemblies, parts or components. An example of hierarchical division is shown in Figure 5.4.

Some insight on decomposition of a wind turbine can be also obtained from available information about failures of different assemblies or components. Taking into consideration the frequency of failures in a given component, decisions can be made about the extent of detailed analysis required to achieve acceptable reliability level (some sources for failure rates are presented in section 3.1).
Figure 5.5. Normalized failure rates of sub-systems and assemblies of wind turbines, (Wilkinson, et al., 2011)

Technology Assessment and Categorization

When dealing with design of new technologies or applying innovative improvements to existing ones, a certain amount of novelty is always introduced. During this phase, following (or similar) questions should be addressed, (Ballesio, et al., 2009)

- Which aspects of the novel technology introduce hazards and pose significant risks?
- Which risks can be dealt with efficiently by design measures?
- What codes or standards could be applied to reduce identified risks?
- What aspects/parts of the novel technology are not covered by current standards, rules and regulations and needs to be qualified?

According to (DNV, 2013), novelty of a technology can be categorized as shown in Table 4. Technology categorization is an important part of Technology Assessment (step 2).

Table 5.1. Technology categorization. Adopted from (DNV, 2013)

<table>
<thead>
<tr>
<th>Application Area</th>
<th>Degree of novelty of technology</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proven</td>
</tr>
<tr>
<td>Known</td>
<td>1</td>
</tr>
<tr>
<td>Limited Knowledge</td>
<td>2</td>
</tr>
<tr>
<td>New</td>
<td>3</td>
</tr>
</tbody>
</table>

(INNWIND.EU, Deliverable 1.34, Integrated system reliability analysis)
Based on the technology categorization, decisions can be made whether to proceed with TQ process for a given component or if it is already well defined and no further TQ efforts are required in order to document sufficient reliability. According to (DNV, 2013), Category 1 components can be assumed “known and proven” and therefore do not require to be included in the TQ process and related reliability can be assessed using existing standards. Category 2-4 are considered “novel” and require further investigation because of increased technical uncertainty.

It is also important to notice that if already proven and qualified components of a system are used in a different environment or assembled in a different way, the system as a whole may require (re)qualification.

Identifying of main technical challenges and uncertainties is a part of Technology Assessment step in TQP. For complex systems identification can be performed by HAZID (Hazard Identification) or HAZOP (Hazard and Operability) analysis. These analyses should be performed in a form of workshops where relevant experts can share their knowledge and identify challenges and uncertainties related to given technological innovation.

**Threat assessment**

The goal of this step of TQP is to identify relevant failure modes and mechanisms of the innovative technology and evaluate associated risks. Based on the frequency of a particular failure and its consequences activities in the Technology Qualification Plan can be prioritized to focus efforts on most severe failures.

Threat assessment can be performed using several well-known methods:

- Failure Mode, Effect and (Criticality) Analysis (FME(C)A).
- Hazard and Operability studies (HAZOP).
- Fault Tree Analysis (FTA).
- Structured What-If checklists (SWIFT).
- Operational Problem Analysis (OPERA).
- Risk Screening (Hazid sessions)
- Independent review by experts.

Depending on allocated recourses and available time, the most appropriate method should be chosen from the ones mentioned above. FMECA, FTA, HAZID and HAZOP are reliable systematic methods, widely used in the industry despite the fact that in most cases they are time consuming. HAZID could be used to perform a global system
analysis and FMECA or FTA could be more specific and focus more on individual subsystems and components (Ballesio, et al., 2009). SWIFT and OPERA require highly qualified experts in order to provide satisfactory results.

The probability of failure for each failure mode during the early development phases should be estimated by field experts. Table 5 shows an example of failure probability classification.

Table 5.2. Generic qualitative probability classes, (DNV, 2013).

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Failure is not expected ( (p_f &lt; 10^{-4}) )</td>
</tr>
<tr>
<td>2</td>
<td>An incident has occurred in industry or related industry ( (10^{-4} &lt; p_f &lt; 10^{-3}) )</td>
</tr>
<tr>
<td>3</td>
<td>Has been experienced by most operators ( (10^{-3} &lt; p_f &lt; 10^{-2}) )</td>
</tr>
<tr>
<td>4</td>
<td>Occurs several times per year per operator ( (10^{-2} &lt; p_f &lt; 10^{-1}) )</td>
</tr>
<tr>
<td>5</td>
<td>Occurs several times per year per facility ( (10^{-1} &lt; p_f) )</td>
</tr>
</tbody>
</table>

The failure probabilities should be based on existing knowledge about similar components used in same or at least comparable environments. Later on, when the qualification progresses, new knowledge from testing and numerical modelling should be used and probabilities of failure can be updated. The use of failure statistics databases can also be useful, when applicable. Another aspect of threat (risk) assessment is failure severity. Table 6 shows an example of failure consequence classification.

Table 5.3. Failure severity classes. Adopted form (Bureau Veritas, 2010) and (JCSS, 2008).

<table>
<thead>
<tr>
<th>F</th>
<th>Severity</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Negligible</td>
<td>No damage to personnel, safety functions fully available</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non significant spill, minor environment impact</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No off-site impact/damage</td>
</tr>
<tr>
<td>2</td>
<td>Minor</td>
<td>Light injuries to personnel and/or local damage to safety functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A few barrels of pollution to sea, moderate environment impact,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Minor off-site impact</td>
</tr>
<tr>
<td>3</td>
<td>Severe</td>
<td>Serious injuries to personnel and/or large local damages to safety functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A few tonnes of pollution to sea, significant environmental impact,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Situation is manageable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Moderate off-site impact limited to property damage or minor health effects</td>
</tr>
<tr>
<td>4</td>
<td>Critical</td>
<td>One fatality, or less than 10 on-site permanent disabling injuries,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Impairment of safety functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Serious environment impact, significant pollution demanding urgent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>measures for the control of the situation and/or cleaning of affected</td>
</tr>
<tr>
<td></td>
<td></td>
<td>areas</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Significant off-site property damage or short term health effects to public</td>
</tr>
<tr>
<td>5</td>
<td>Catastrophic</td>
<td>Multiple fatalities and/or 10 or more on-site permanent disabling</td>
</tr>
<tr>
<td></td>
<td></td>
<td>injuries also outside the event area, total</td>
</tr>
<tr>
<td></td>
<td></td>
<td>impairment of safety functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Extensive environment impact, Major pollution with difficult control of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>situation and/or difficult cleaning of affected areas</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Extensive off-site property damage, fatalities or short term health</td>
</tr>
<tr>
<td></td>
<td></td>
<td>effects to public</td>
</tr>
</tbody>
</table>
Having failure probabilities and failure consequences defined it is possible to prioritize the failures using Risk Matrix.

![Risk Matrix Diagram](image)

**Figure 5.6.** Risk matrix. L – low risk, M – medium risk H – high risk. *(DNV, 2013)*

According to *(DNV, 2013)* failure modes with medium and high risk are considered *critical* and should be considered in TQP. Failure modes with low risk can be accounted for by qualitative assessment of qualified personnel, however low risk failure modes should not be omitted from the list of possible failures. Another way of prioritizing failures is based on Risk Priority Number (RPN). It can be calculated using the following equation:

\[ RPN = S \cdot P \cdot D \]  

(5.1)

where:
- S – failure severity class;
- P – failure probability class;
- D – failure detection rating.

The detection rating depends on the possibility to detect failures before they happen. Table 7 shows an example of detection scale.
Technology Qualification plan

Technology Qualification plan is developed to provide the evidence needed to manage the critical failure modes defined during Treat Assessment. In Technology Qualification basis the target reliability of systems and components is defined in order to eliminate high risk failure modes or reduce the risks to acceptable levels. Therefore quantitative reliability methods are required to document components compliance to initial reliability requirements. For each failure mode appropriate failure mechanism models should be used, physical models are preferred where applicable. According to (DNV, 2013) the following methods can be used to provide qualification evidence:

- Analysis/ engineering judgment of previous, documented experience with similar equipment and operating conditions.
- Analytical methods such as handbook solutions, methods from existing standards, empirical correlations or mathematical formulas.
- Numerical methods, such as process simulation models, CFD, FEM, corrosion models, etc.
- Experimental methods.
- Inspections to ascertain that specifications are complied with or assumptions valid.
- Development of new or modified QA/QC requirements for manufacturing / assembly.
- Development of requirements to inspection, maintenance and repair.
- Development of spares policy.
- Development of operating procedures resulting from the Technology Qualification Process.

Parameter effects, numerical modelling

The qualification process has to take into account the effects of uncertainties in critical parameters. If a proven numerical model, simulating the failure mechanism in the intended environment, exits, it can be used to assess parameter effects associated with that failure mechanism. If available models are lacking confidence (i.e. models are developed for different environmental conditions), they should be verified by testing in...
the relevant parameter ranges for short and long-term behavior. If no proven numerical modes exist for a particular failure mechanism, the following mode-based qualification approach should be used, (DNV, 2013):

- Utilize or develop models of the critical failure modes using, to the extent possible, models that have been proven.
- Identify uncertainties in the models and model input data.
- Characterize input data for models with the associated uncertainty.
- Build confidence in the models. This is achieved through qualification activities that challenge them by comparing model predictions with relevant evidence, e.g. from experiments performed as part of the qualification, published data or service experience.
- Use model to quantify parameter effects.

In situations where no model can be developed, the parameter effects can be assessed by qualification tests or, to some extent, by conservative expert judgment.

Experimental methods

The purpose of experimental testing, according to (DNV, 2013) is:

- Explore novel elements of the technology, and help identify failure modes and mechanisms of relevance.
- Provide empirical evidence for functionality and reliability assessment.
- Identify or verify the critical parameters and their sensitivities.
- Characterize input parameters for models.
- Challenge model predictions of failure mechanisms. To challenge the models, the tests should cover extreme values of the critical parameters with respect to the qualification basis.

Characterization testing helps to determine the input for qualification models, when there is no proven and accepted literature (i.e. when new materials are used and material properties and their uncertainties are unknown).

Component and prototype testing in the early stages of qualification process are helping to explore the new elements of the technology, identify relevant and new failure modes/mechanisms. In early stages testing to failure is preferred when possible due to ability to explore the critical parameters and their effects close and at failure.

Another reason for prototype and component testing is numerical model validation and challenging. Test results and model predictions can be compared in order to verify if numerical models are applicable. If the testing environment represents the intended
service environment close enough, then, given that the tests produce statistically significant estimates of systems behavior, such tests can be regarded as sufficient to provide direct evidence about the behavior of the system. Since prototype testing is very costly, usually tests are used only for model verification and, if verification is successful, the model is considered as qualification evidence.

**Execution of TQ plan**

Main activities while executing the TQ plan are related to collecting the data from numerical/analytical modelling, verification and prototype testing and ensuring that the data is easily traceable. Determining performance margins for each failure mode is also important. While executing the TQ plan failure modes will be detected. When a failure is detected, according to (DNV, 2013), it will be evaluated with respect to the three following cases:

- Failure mode occurred within the expected frequency of occurrence according to the analysis.
- Failure mode occurred with a higher frequency of occurrence.
- Failure mode has not been considered.

In first case no action is needed. In second case the assumptions about the frequency of the failure mode have to be re-evaluated. In the third case the failure mode has to be taken into consideration, this resembles to the iterative nature of the TQP.

**Performance assessment**

The purpose of performance assessment is to measure the compliance of qualification evidence to the requirements presented in the Technology Qualification Basis (TQB). This implies checking if the TQB requirements are met within acceptable limits of uncertainty. According to (DNV, 2013), the key steps of performance assessment are:

- Interpret the evidence in the specific context of the technology, to account for simplifications and assumptions made when the evidence was generated, and limitations and approximations in the methods used.
- Confirm that the qualification activities have been carried out, and that the acceptance criteria have been met. A key part of this confirmation is to carry out a gap analysis to ensure that the qualification evidence for each identified failure mode meets the acceptance criteria.
- Perform a sensitivity analysis of relevant parameter effects.
- Assess the confidence that has been built into the qualification evidence through the qualification activities. This shall consider the extent to which
• Compare the failure probability or performance margin for each identified failure mode of concern with the requirements laid down in the Technology Qualification Basis. Evidence shall be propagated for individual elements of novel technology and reviewed against the entire system covered by the Technology Qualification.

If TQB requirements are expressed in terms of target reliability, quantitative reliability assessment has to be carried out. For time dependent failures, the expected lifetime or time to maintenance will be determined and compared to TQB requirements. System reliability can be estimated by:

• Reliability block diagram technique (RBD), which considers system with components in series and parallel.
• Fault tree analysis (FTA), which considers combinations of subsystems, lower-level faults and component faults. FTA is a top-down analysis, and therefore has to be repeated for each top event.
• Monte Carlo simulation of RBDs, FTs or more complex systems utilizing some suitable software tool.
6. SUPPORTIVE TOOLS AND METHODOLOGIES FOR TQP

6.1. Reliability block diagrams

The Reliability Block Diagram or RBD is a representative drawing and a calculation tool that is used to model system availability and reliability. The structure of a reliability block diagram defines the logical interaction of failures within a system and not necessary their logical or physical connection together. Each block can represent an individual component, sub-system or other representative failure. The diagram can represent an entire system or any subset or combination of that system which requires failure, reliability or availability analysis. It also serves as an analysis tool to show how each element of a system functions, and how each element can affect the system operation as a whole. Figure 6.1 shows a simplified RBD for a wind turbine.

A series system RDB can be solved by multiplying the reliabilities of separate assemblies or sub-assemblies. A more complex RBD involving redundancy (parallel systems) can be solved using rules of probability.

![Figure 6.1. Simplified reliability diagram of wind turbine.](image)

6.2. Failure mode analysis

Failure mode and effect (criticality) analysis is a design tool that provides means to identify and evaluate potential design and process failures before they occur, with the purpose of eliminating them or minimising the risk related to them (IMCA, 2002). Also, FME(C)A can be used to allow for improvements in system reliability and maintainability by highlighting the areas where design modifications are necessary or there is unconformity for maintainability. Typically FMEA is of qualitative nature and FME(C)A is quantitative. Within FME(C)A framework, the individual failure modes and their effects on the system are treated independently, therefore the procedure is not suitable for failures resulting from a sequence of events. To analyse these types of failures other methods should be used – Markov or Fault Tree analyses (CENELEC, 2006).
FME(C)A is usually performed in bottom-up fashion following the hierarchical decomposition of the system from lowest (component) level, where the potential failure modes are known and investigating their effects on the next (sub-system) level.

The process of carrying out an FME(C)A study is as follows, based on (Pillay A., 2003):

1. Develop a good understanding of what the system is supposed to do when it is operating properly.
2. Divide the system into sub-systems and/or assemblies in order to ‘localise’ the search for components.
3. Use blue prints, schematics and flow charts to identify components and relations among components.
4. Develop a complete component list for each assembly.
5. Identify operational and environmental stresses that can affect the system. Consider how these stresses might affect the performance of individual components.
6. Determine failure modes of each component and the effects of failure modes on assemblies, sub-systems, and the entire system.
7. Categorise the hazard level (severity) of each failure mode (several qualitative systems have been developed for this purpose).
8. Estimate the probability. In the absence of solid quantitative statistical information, this can also be done using qualitative estimates.
9. Calculate the risk priority number (RPN).
10. Determine if action needs to be taken depending on the RPN.
11. Develop recommendations to enhance the system performance.
12. Summarise the analysis: this can be accomplished in a tabular form.

FME(C)A is mostly a qualitative method of hazard identification, although, if RPN (Risk Priority Number) is used, it can be extended to provide a good measure of risk, associated to any given failure. RPN can be calculated as described in section 5.1. Furthermore, Cost-Priority-Number (CPN) can be used to better quantify the overall impact of wind turbine failures in monetary terms (Shafiee M., 2014). If cost information for components, involved in failure mode is available, CPN can be calculated as follows (for further detail refer to (Shafiee M., 2014)):

\[
CPN = O \cdot C \cdot D
\]  

where:
- \(O\) probability of occurrence that can be obtained from field data.
- \(C\) cost consequence of failure, including all costs related to given failure (spare parts, replacement crew, production loss etc.).
- \(D\) non-detection possibility which is a ratio between actual failures and number of failure opportunities (sum of actual failures and prevented failures).

A typical format for FME(C)A report can be seen in Figure 6.3. General guidelines for performing an FME(C)A study can be found in (CENELEC, 2006) standard.

<table>
<thead>
<tr>
<th>System</th>
<th>FMEA No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem</td>
<td>Page</td>
</tr>
<tr>
<td>Component</td>
<td>Prepared by</td>
</tr>
<tr>
<td>Core team</td>
<td>FMEA Date (org.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component/ process</th>
<th>Potential failure mode</th>
<th>Potential effects of mode</th>
<th>Potential causes of mode</th>
<th>Present control mechanisms</th>
<th>Severity</th>
<th>Occurrence</th>
<th>Detection</th>
<th>Risk priority number (RPN)</th>
<th>Recommend actions</th>
<th>Action taken</th>
<th>5</th>
<th>0</th>
<th>D</th>
<th>RPN</th>
</tr>
</thead>
</table>

Figure 6.3. Typical FME(C)A report, adopted from (Liu, et al., 2011).

6.3.Fault Tree Analysis

A Fault tree is a logical diagram that displays the interrelationships between a potential critical event in a system and the causes for this event. Causes can be of different origin – environmental conditions, human error, normal operational events, specific component failures, etc. This method focuses on a single system failure mode and can provide information on how a particular event can occur, what consequences it leads to and what system components are involved in failure process (Zio, 2007). FTA can be both quantitative and qualitative therefore it is a very useful tool for reliability assessment of a
system and also for technology qualification. Depending on the nature of FT Analysis the results may include (Rausand & Hoyland, 2004):

- A list of all possible combinations of causes that result critical events (system failures), including component failures, human errors, etc. This is achieved by qualitative FTA.
- The probability of critical event occurrence within a specific time interval. This can be achieved by quantitative FTA.

FTA for complex systems can be started by constructing a system flow diagram (usually Reliability Block Diagrams (RBD) are used) in order to depict how the materials and/or energy are transmitted between the components of the system. Having RBD allows starting FT analysis, which can be carried out in 5 basic steps (Rausand & Hoyland, 2004):

**Step 1: Definition of the problem and the boundary conditions.**

This part involves the definition of a critical event/failure (TOP event) and a description of the type of the event (eg. Blade failure). A description of where and when the event is occurring should also be included when possible (eg. at blade root during normal operation/extreme wind conditions). The definition of boundary conditions should be understood as: the physical boundaries (which parts of the system are involved in event), initial conditions of the system (is the system in normal operation, shut-down or start-up setup), boundary conditions with respect to external environmental effects (eg. normal operating conditions, extreme wind/wave conditions), the level of resolution (how detailed the analysis will be, e.g. blade failure due to pitch system fault – should the pitch system be analysed separately or included in blade failure fault tree).

**Step 2: Construction of the fault tree.**

The fault tree construction always starts with a TOP event that is followed by immediate events (basic or secondary, for definitions refer to (Rausand & Hoyland, 2004) Chapter 7) that causes the top event. TOP events are connected to initiating, lower level events by AND (top event occurs if all inputs occur) or OR gates (top event occurs if only one input occurs). Figure 6.4 shows an example of complex FT for gearbox. Basic events causing the top event can be failures of the device itself (due to aging or extreme external effects), human errors in installing or actuating the device, no input to the device (eg. failure in electrical control circuits).
For more details on construction of the FTs refer to (Rausand & Hoyland, 2004). More information is provided in (IEC, 2007) standard, where standardised descriptions of static gates are provided in Appendix A.

**Step 3: Identification of minimal cut sets.**

If more than one basic event is contributing in initiating a TOP event fault, minimal cut sets should be found. A minimal cut set is a set of basic events ensuring that the TOP event occurs.

**Step 4: Qualitative analysis of a fault tree.**

A qualitative analysis of a fault tree is usually carried out on the basis of minimal cut sets. A cut set of order 2 (two events have to occur to facilitate a TOP event) is less critical than a set of order 1 (only one event has to occur for TOP event to happen). Therefore the order of cut sets for any given failure can be used to assign criticality to different failures of a system. Qualitative analysis also involves evaluation of different factors facilitating the basic events (human errors in handling the devices, control signal errors, active equipment failure, etc.). Depending on these factors, measures that reduce the risk can be taken – reducing the chance of human errors, designing a more reliable control system or using more reliable primary equipment.
Step 5: Quantitative analysis of a fault tree.

Quantitative FTA consists of transforming the logical structure into an equivalent probability form and numerically calculating the probability of occurrence of the TOP event. This is done by proceeding from bottom to top and evaluating each AND/OR gate separately using basic laws of probability (case of two independent input items):

\[
P(TOP_{\text{event}}) = P_1 P_2 \quad \text{for AND gates;}
\]

\[
P(TOP_{\text{event}}) = P_1 + P_2 - P_{12} \quad \text{for OR gates;}
\]

6.4. HAZOP

Hazard and Operability studies are based on a theory that assumes risk events are caused by deviations from design or operating intentions. HAZOP is a bottom-up hazard identification methodology that examines processes, allowing for identification of the initiating events of undesired accident/failure sequence. It identifies all possible hazards associated with accidents/failures and determines the resultant effects. The effects have to be analysed in detail so that the risk of events can be quantified in tangible terms (likelihood and consequences should be assessed). The analysis procedure is as follows (Zio, 2007):

1. Decompose the system in functionally independent process units; for each unit identify the various operation modes (start-up, shut-down, maintenance).
2. For each process unit and operation mode, identify the potential deviations from normal behaviour:
   a. Specify all the unit incoming and outgoing fluxes (energy, control signals, fluids, etc.);
   b. Write down the various functions that unit is supposed to attend (generate electricity, provide pressure etc.);
   c. Apply keywords such as low, high, no, reverse, etc., to the identified process variables and unit functions, so as to generate deviations from the normal regime.
3. For each process deviation, qualitatively identify its possible causes and consequences (quantitative evaluation of consequences should be performed if possible). For the consequences, include effects also on other units, this allows HAZOP to account also for domino effects among different units.

HAZOP and FME(C)A analyses should be used together in order to maximise the number of failure modes considered in design and design verification process. The flowchart in Figure 6.5 presents the general sequence of performing a HAZOP study. General guidelines can be found in (IEC, 2001) standard.
6.5. Pareto analysis

Pareto analysis is a tool used to rank categories in the descending order of occurrence to separate significant categories from trivial ones. Pareto analysis should be performed after RBD and FTA analysis is done and failure probabilities/frequencies are established for different components of a system.

Figure 6.5. HAZOP flowchart, adopted from (IEC, 2001).
6.6. Accelerated testing

While developing products and technologies that require a certain level of reliability, Accelerated Testing (AT) is commonly used in order to identify and assess possible failures in reasonably short time. Depending on the type of the device, AT conditions may involve higher level of stress, vibration, temperature, voltage, etc. compared to usual levels during normal or extreme operation (Zio, 2007). Generally AT can be divided into two types – qualitative and quantitative.

Qualitative AT is used in early development stages in order to reveal possible failure modes that can be eliminated afterwards. Environmental Stress Screening (ESS) is a type of Qualitative AT where rapid environmental stimuli (temperature, vibration) are applied to electrical components in order to precipitate latent defects into observable failures. It has to be noted, that ESS is not a simulation of normal operation of the device, ESS is designed to apply high magnitude stimulation that reveals flaws in the design. Furthermore, it is important to ensure that the applied stresses are not exceeding the stress limits of any components and irrelevant failure modes are triggered (Dinesh Kurnar, 2006). More information on ESS is available in (Dinesh Kurnar, 2006) Chapter 5.

Quantitative AT can be divided into two main groups, namely usage rate acceleration (number of load cycles is increased, the stress frequency is increased) and over-stress acceleration (stress intensity is increased).
Accelerated Stress Testing (AST)

Figure 6.7 shows a summary of different strategies when performing AST. Left side represents testing according to commonly accepted standard requirements against constant failure rate models, which do not always represent failures in the field. The right side represents performing tests that are specifically tailored for the specific failure modes of the system based on knowledge of failure mechanisms encountered in the field.

When performing AST one strategy is to increase the severity of real but extreme stress that would be close to operating limits of the component in question given that the failure mechanism remains the same in reality and the test (Figure 6.8, left). Another strategy is to increase the probability of extreme stresses that the component is subjected to (Figure 6.8, right). A third strategy would be to reduce the strength (increased susceptibility stress) of the component so that normal stresses act like extreme stress. However, this type of testing is very difficult to design and perform and is still in research stages only (Yuan Lu, 2000). Generally for all AST strategies a good understanding of failure mechanisms and stress distributions is necessary for the tests to produce reliable results.
Highly Accelerated Stress Testing (HAST)

HAST testing involves increasing a single stress step by step until a failure occurs. The risk of performing HAST is that irrelevant failure mechanisms can be activated and therefore it is a challenge to assure that the relevant failure mechanism is simulated. Although, given that only relevant failure modes are activated, HAST can produce good lifetime estimates in a short timeframe. Another drawback is that HAST testing only involves one stress at a time and possible interactions between stresses that are observed in service are ignored.

6.7. Updating of probabilistic models by integrating test results

Throughout the development process testing should be conducted in order to reduce the model uncertainty. In general, testing at different scales and component complexity should be concluded:

- Coupon tests with basic material and measurement of climatic parameters at an early design stage can be conducted and the information can be used to update the physical variables (Sørensen & Toft, 2010). This stage of testing involves tests of materials and basic parts and components at lowest complexity level.
- Sub component testing involves tests of more complex components such as flanged connections or beams of wind turbine blades.
- Testing of full scale components such as blades, generators or gearboxes.

Figure 6.9 shows the extent of testing at each scale and illustrates the changes in associated model uncertainty.
A testing plan should be developed in preparatory stages of the development and followed later-on. V-Model is a good reference when it comes to test planning in technology development.

The probabilistic models can be updated with the testing information by using Bayesian methods. More information on Bayesian updating can be found in Appendix D.
7. EXAMPLE - RELIABILITY ASSESSMENT OF MRS SUPPORT STRUCTURE

The section describes an overall reliability assessment of the support structure for the Multi Rotor System (MRS) described in Deliverable 1.33. The reliability analyses are based on the utilization ratios of the structural members of the support structure derived in D1.33 coupled with the limit states / failure modes described in D1.33 and the stochastic models used for calibration of material partial safety factors for the new edition 4 of IEC 61400-1 as described in the background document (Sørensen & Toft, 2014).

The following generic limit state equation the extreme load effect in operation (DLC 1.1) or standstill (DLC 6.1) are described in (Sørensen & Toft, 2014)

\[ g = z \eta \delta R - X_{\text{dyn}}X_{\text{exp}}X_{\text{aero}}X_{\text{str}}L \]  

(7.1)

where
- \( z \) design parameter, e.g. cross-sectional area
- \( \eta \) utilization ratio
- \( \delta \) model uncertainty load bearing model
- \( R \) uncertainty in dominating strength parameter
- \( X_{\text{dyn}} \) uncertainty related to modeling of the dynamic response, including uncertainty in damping ratios and eigenfrequencies
- \( X_{\text{exp}} \) uncertainty related to the modeling of the exposure (site assessment) - such as the terrain roughness and the landscape topography
- \( X_{\text{aero}} \) uncertainty in assessment of lift and drag coefficients and additionally utilization of BEM, dynamic stall models, etc.
- \( X_{\text{str}} \) uncertainty related to the computation of the load-effects given external load
- \( L \) uncertainty related to the extreme load-effect due to wind loads

The corresponding design equation is written:

\[ \frac{z \eta R_k}{\gamma_M} - \gamma_f L_k \geq 0 \]  

(7.2)

where
- \( R_k \) characteristic value of load bearing capacity
- \( L_k \) characteristic value of variable load
- \( \gamma_M \) partial safety factor for load bearing capacity
- \( \gamma_f \) partial safety factor for load effect

Two failure modes are considered: failure by yielding (for members in tension) and stability / buckling failure (for members in compression).

For failure by yielding the load bearing capacity is proportional with the yielding strength and model uncertainty depending on the complexity of the member. Based on information from
(Sørensen & Toft, 2014) and (JCSS, 2002) representative stochastic models for yielding strength, \( R \) and the model uncertainty, \( \delta \) are:

\[
R \sim \text{LogNormal with coefficient of variation (COV) } = 0.05 \\
\delta \sim \text{LogNormal with coefficient of variation (COV) } = 0.05 \text{ and mean value (bias) } = 1
\]

For failure by buckling a representative stochastic model is derived in (Sørensen & Toft, 2014) coupled to local buckling failure and applying for deterministic design the parametric formulas based on membrane theory in Eurocode 3 part (EN 1993-1-6, 2006) for shell buckling applicable to tubular members with Diameter/Thickness < 300.

As described in (Sørensen & Toft, 2014) the buckling strength of shells has been studied in several test programs where cylindrical shells are loaded in axial compression. Figure 7.1 shows test results from the literature compared to the buckling curves used in Eurocode 3. From the figure it is seen that the buckling reduction factor \( \chi_x \) contains a significant uncertainty dependent on the relative slenderness \( \lambda_x \). It is also seen that the buckling curves used in Eurocode 3 are not specified as mean curves and the bias introduced by using these buckling curves should therefore be taken into account in the reliability assessment.

![Figure 7.1. Axial compression cylinder tests compared to the Eurocode 3 buckling curves, (ECCS, 2008)](image-url)
Figure 7.2 shows the mean buckling curve based on the test data shown in Figure 7.1. As described in (Sørensen & Toft, 2014) the following simple, representative stochastic models for yielding strength, $R$, and the model uncertainty, $\delta$, can be applied:

- $R$ LogNormal with coefficient of variation (COV) = 0.05 and characteristic value (5% quantile) = 360 MPa
- $\delta$ LogNormal with coefficient of variation (COV) = 0.13 and mean value (bias) = 1/0.85

In D1.33 the Design Load Cases DLCs) 1.3 and partly 6.2 are considered. The load effects obtained are considered to be equivalent to those obtained from DLC 1.1 (extreme load during operation) and DLC 6.1 (extreme when the turbine is parked). The stochastic model in Table 7.1 (from (Sørensen & Toft, 2014)) is used as ‘representative’ for the reliability analyses of the MRS, see [16].

**Table 7.1. Stochastic models for physical, model and statistical uncertainties.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>COV</th>
<th>Quantile</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Lognormal</td>
<td>-</td>
<td>$V_R$</td>
<td>5%</td>
<td>Strength</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Lognormal</td>
<td>-</td>
<td>$V_\delta$</td>
<td>Mean</td>
<td>Model uncertainty</td>
</tr>
<tr>
<td>$L$ – DLC 1.1 / 1.3</td>
<td>Weibull</td>
<td>-</td>
<td>0.15</td>
<td>0.98</td>
<td>Annual maximum load effect obtained by load extrapolation</td>
</tr>
<tr>
<td>$L$ – DLC 6.1 / 6.2</td>
<td>Gumbel</td>
<td>-</td>
<td>0.2</td>
<td>0.98</td>
<td>Annual maximum wind pressure</td>
</tr>
<tr>
<td>$X_{dyn}$</td>
<td>Lognormal</td>
<td>1.00</td>
<td>0.05</td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>$X_{exp}$</td>
<td>Lognormal</td>
<td>1.00</td>
<td>0.15</td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>$X_{aero}$</td>
<td>Gumbel</td>
<td>1.00</td>
<td>0.10</td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>$X_{str}$</td>
<td>Lognormal</td>
<td>1.00</td>
<td>0.03</td>
<td>Mean</td>
<td></td>
</tr>
</tbody>
</table>
The following partial safety factors are used:

\( \gamma_M \) partial safety factor for load bearing capacity
- \( \gamma_M = 1.1 \) for yielding failure mode and
- \( \gamma_M = 1.1 \) for stability / buckling failure mode

\( \gamma_f \) partial safety factor for load effect (turbine loads)
- \( \gamma_f = 1.35 \) for DLC 1.1 / 1.3 and
- \( \gamma_f = 1.35 \) for DLC 6.1 / (6.2)

For the yielding failure mode the utility ratio can be obtained from

\[
\eta = \frac{S_{\text{max}}}{R_k / \gamma_M}
\]  

(7.3)

where
- \( R_k = 360 \text{MPa} \) = characteristic value of load bearing capacity
- \( S_{\text{max}} \) design maximum load effect in a given MRS structural member obtained from Figure 6.8-1 and 6.8-2 in D 1.33, see also figure below
- \( \gamma_M \) = 1.1 – partial safety factor

For the stability / buckling failure mode the utility ratios are shown in Figure 6.8-1 and 6.8-2 in D1.33, see figures below.

In the following the reliability is estimated for each structural member in the MRS for the above failure modes and load cases. The reliability is expressed by the annual reliability index \( \beta \) obtained by FORM (First Order Reliability Method), see (Madsen, et al., 1986) and (Sørensen, 2011). Besides the reliability index also the so-called \( \alpha \)-vector is obtained which is a unit vector indicating the importance of each stochastic variable.

Since the MRS consists of 275 structural members and system failure / total collapse / major damage may occur if one of the structural members fail, the system reliability is estimated considering a series system model consisting of potential failure in any of the structural members as elements in the series system model. It is noted that some additional load bearing capacity may exist in case of failure of a structural member. Assessment of this additional resistance requires non-linear finite element analysis which is outside the scope of this investigation.

The system probability of failure for a series system with \( m \) elements is estimated by
\[ P_f^S = P\left( \bigcup_{i=1}^{m} \{ g_i(X) \leq 0 \} \right) \approx 1 - \Phi_m(\beta; \rho) \] (7.4)

where \( \Phi_m(.) \) is the standardized \( m \)-dimensional Normal distribution function, \( \beta \) is the vector with reliability indices and the elements in the correlation matrix, \( \rho \) are obtained from \( \rho_{ij} = \alpha_i' \alpha_j \).

The \( \alpha \)-vector \( \alpha_j \) is obtained as

\[ \alpha_j = \left( 0, 0, \ldots, \alpha_{R_j}, 0, 0, \ldots, 0, \alpha_\delta, \alpha_L, \alpha_{X_{\text{exp}}}, \alpha_{X_{\text{str}}}, \alpha_{X_{\text{str}}} \right) \] (7.5)

where the index indicates which stochastic variable the \( \alpha \)-value is connected to. It is noted that the strength \( R \) for different structural members are assumed statistically independent.

The system reliability index is defined as

\[ \beta_s = -\Phi^{-1}(P_f^S) \] (7.6)

In the following figures utility ratios and reliability indices are shown for
- the yielding failure mode for DLC 6.1/6.2, see Table 7.3 and 7.4
- the stability / buckling failure mode for DLC 6.1/6.2, see Table 7.5 and 7.6
- the yielding failure mode for DLC 1.1/1.3, see Table 7.7 and 10.8
- the stability / buckling failure mode for DLC 1.1/1.3, see Table 7.9 and 7.10
For structural member no. 58 the reliability analysis results in an annual reliability index $\beta = 2.76$ and $(\alpha_R, \alpha_\delta, \alpha_L, \alpha_{X_{\text{dyn}}}, \alpha_{X_{\text{exp}}}, \alpha_{X_{\text{aero}}}) = (-0.21, -0.21, 0.41, 0.21, 0.61, 0.56, 0.12)$ indicating that the most important uncertainties are related to $X_{\text{exp}}$ and $X_{\text{aero}}$. The element correlation
coefficients are approximately 0.97 and the system reliability index becomes $\beta_S = 2.58$. It is seen, that several members have a reliability index close to the lowest reliability index, but due to the high correlation between the elements the system reliability is still relatively high, but lower than the target annual reliability index $= 3.5$ in (Sørensen & Toft, 2014).

Figure 7.5. $\eta$ for stability / buckling failure for DLC 6.1/6.2 for all 275 structural members.

Figure 7.6. Annual reliability index, $\beta$ for the stability / buckling failure mode for DLC 6.1/6.2 for all 275 structural members.
For structural member no. 58 the reliability analysis results in reliability index $\beta = 3.58$ and $(\alpha_R, \alpha_\delta, \alpha_L, \alpha_{\text{exp}}, \alpha_{\text{aero}}, \alpha_{\text{w}}) = (-0.18, -0.47, 0.35, 0.18, 0.54, 0.53, 0.11)$ indicating that the most important uncertainties are related to $\delta$, $X_{\text{exp}}$ and $X_{\text{aero}}$. The element correlation coefficients are approximately 0.96 and the system reliability index becomes $\beta_S = 3.49$. It is seen, that several members have a reliability index close to the lowest reliability index, but due to the high correlation between the elements the system reliability is still relatively high and close to the target annual reliability index $= 3.5$ in (Sørensen & Toft, 2014).

![Figure 7.7. S_{max} (in MPa) for DLC 1.1/1.3 for all 275 structural members.](image)
Figure 7.8. Annual reliability index, $\beta$ for the yielding failure mode for DLC 1.1/1.3 for all 275 structural members.

For structural member no. 245 the reliability analysis results in reliability index $\beta = 4.28$ and $(\alpha_\eta, \alpha_\delta, \alpha_L, \alpha_{X_{exp}}, \alpha_{X_{syn}}, \alpha_{X_{aero}}) = (-0.19, -0.19, 0.35, 0.19, 0.57, 0.65, 0.12)$ indicating that the most important uncertainties are related to $X_{exp}$ and $X_{aero}$. The element correlation coefficients are approximately 0.97 and the system reliability index becomes $\beta_s = 4.27$. It is seen, that only few members have a reliability index close to the lowest reliability index. Therefore the system reliability is almost equal to the lowest element reliability index and larger than the target annual reliability index $= 3.5$ in (Sørensen & Toft, 2014).
Figure 7.9. $\eta$ for stability / buckling failure for DLC 1.1/1.3 for all 275 structural members.

Figure 7.10. Annual reliability index, $\beta$ for the stability / buckling failure mode for DLC 1.1/1.3 for all 275 structural members.

For structural member no. 245 the reliability analysis results in reliability index $\beta = 4.27$ and $(\alpha_R, \alpha_\beta, \alpha_L, \alpha_{X_{dyn}}, \alpha_{X_{exp}}, \alpha_{X_{max}}, \alpha_{X_{w}}) = (-0.18, -0.46, 0.33, 0.18, 0.53, 0.57, 0.11)$ indicating that the
most important uncertainties are related to $\delta$, $X_{\text{exp}}$ and $X_{\text{aero}}$. The element correlation coefficients are approximately 0.96 and the system reliability index becomes $\beta_S = 4.27$. It is seen, that only few members have a reliability index close to the lowest reliability index and larger than the target annual reliability index $= 3.5$ in (Sørensen & Toft, 2014).

In summary the above reliability assessments of the structural system reliability of the MRS support structure show that:

- for the failure modes related to DLC 1.1/1.3 (extreme turbulence in operating mode) the support structure is structurally optimized resulting in many members with reliability close to the lowest reliability index
- for the failure modes related to DLC6.1/6.2 only very few elements have reliability index close to the lowest reliability index
- generally the model uncertainties related site assessment and aeroelastic models ($X_{\text{exp}}$ and $X_{\text{aero}}$) are the most important uncertainties due to their relative high coefficient of variations. Therefore improving the accuracy of site assessment and the aerodynamic models has the potential to increase the reliability significantly. The high importance of $X_{\text{exp}}$ and $X_{\text{aero}}$, and the because the model uncertainties are fully correlated for all failure modes imply high correlation between the elements (failure modes) in the systems model.
- due to high correlation between the elements the system reliability index is only slightly lower than the lowest reliability index
- due to a high deterministic utility ratio for the members in the yielding failure mode got DLC 6.1/6.2 the resulting reliability is lower than the target reliability index $= 3.4$ in (Sørensen & Toft, 2014) which is also included in annex K in the draft version of the new IEC 61400-1 ed. 4 standard.
8. EXAMPLE - OPERATION AND MAINTENANCE ASPECTS OF MULTI-ROTOR SYSTEM

8.1. Introduction

This section describes an investigation of the effect of the reliability of the components in the Multi-Rotor System on the availability using a ‘simple’ corrective O&M strategy and a categorization of failures corresponding to the categories used in tools for optimizing an overall O&M strategy, see e.g. (Dinwoodie, et al., 2015). The influence on the availability for the 20MW MRS is compared with a case where two DTU 10MW reference wind turbines are used. It is noted that no cost considerations are included in the results, but some general comments are included.

8.2. General simulation assumptions and set-up

General assumptions regarding site, turbines, weather conditions and O&M strategy:
- Site is chosen at FINO3 ~50km offshore from port of Esbjerg (1.5-2 hour transfer time for a CTV (mall boat), FSV (medium boat) or HLV (Jack-up vessel)).
- Unlimited supply of technicians, vessels and spare parts is considered at this point (no repair scheduling).
- 12 hour working shifts are assumed.
- Weather data from FINO3 is used to generate weather windows and calculate expected power production.
- Available FINO3 wind speed measurements include only variation by height, and no horizontal variation, therefore a horizontally uniform wind field is used to calculate expected power production of the MRS.
- Simulating 25 years of lifetime, with 0.5h time resolution (based on wind speed/wave height data temporal resolution).
- Two DTU 10MW reference turbines are used as baseline for comparison.
- 45x444kW=20MW Multi-Rotors system is used.
- Corrective maintenance is assumed for DTU reference and MRS turbines.
- No costs are included at this point.
- Annual failure rates are grouped into “severity” groups; individual components are not considered (see Table 8.1 and Table 8.2.), (Hendriks, 2015).
- Failures are exponentially distributed.
Table 8.1. Annual failure rates, INNWIND.EU 10MW DTU RWT. (Hendriks, 2015)

<table>
<thead>
<tr>
<th>Component</th>
<th>Sub-component</th>
<th>Cat 1</th>
<th>Cat 2</th>
<th>Cat 3</th>
<th>Cat 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gearbox</td>
<td>Bearings</td>
<td>0.508</td>
<td>0.145</td>
<td>0.067</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>Cooling system</td>
<td>0.068</td>
<td>0.217</td>
<td>0.022</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Gears</td>
<td>0.136</td>
<td>0.011</td>
<td>0.054</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Housing</td>
<td>0.011</td>
<td>0.007</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Lubrication system</td>
<td>0.054</td>
<td>0.264</td>
<td>0.012</td>
<td>0.002</td>
</tr>
<tr>
<td>Generator</td>
<td>Bearings</td>
<td>0.112</td>
<td>0.000</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Windings</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Insulation</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Cooling system</td>
<td>0.080</td>
<td>0.040</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Rotor</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Auxiliaries</td>
<td>0.060</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Structural</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Main shaft set</td>
<td>High speed side</td>
<td>0.005</td>
<td>0.025</td>
<td>0.005</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Main shaft and connections</td>
<td>0.000</td>
<td>0.019</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Main shaft set</td>
<td>Main shaft bearings</td>
<td>0.020</td>
<td>0.147</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Mechanical brake</td>
<td>0.050</td>
<td>0.040</td>
<td>0.015</td>
<td>0.008</td>
</tr>
<tr>
<td>Control &amp; communication system</td>
<td>Controller and communication lines</td>
<td>1.600</td>
<td>0.153</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Safety chain</td>
<td>0.500</td>
<td>0.020</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Auxiliary electrical system</td>
<td>-</td>
<td>0.100</td>
<td>0.150</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Power electrical system</td>
<td>Measurement and cabling</td>
<td>0.280</td>
<td>0.020</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Switchgear</td>
<td>0.200</td>
<td>0.300</td>
<td>0.035</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Transformer</td>
<td>0.170</td>
<td>0.050</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>Frequency converter</td>
<td>Power electronics and control</td>
<td>1.760</td>
<td>0.002</td>
<td>0.105</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Converter cooling system</td>
<td>0.110</td>
<td>0.150</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Hydraulics</td>
<td>-</td>
<td>0.200</td>
<td>0.020</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td>Yaw system</td>
<td>Yaw bearing</td>
<td>0.102</td>
<td>0.012</td>
<td>0.005</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>Yaw brake</td>
<td>0.358</td>
<td>0.080</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Yaw drive</td>
<td>0.560</td>
<td>0.167</td>
<td>0.019</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 8.1 continued. Annual failure rates, 10MW DTU RWT. (Hendriks, 2015)

<table>
<thead>
<tr>
<th>Component</th>
<th>Sub-component</th>
<th>Cat 1</th>
<th>Cat 2</th>
<th>Cat 3</th>
<th>Cat 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nacelle auxiliaries</td>
<td></td>
<td>0,100</td>
<td>0,250</td>
<td>0,000</td>
<td>0,000</td>
</tr>
<tr>
<td>Pitch system</td>
<td>Pitch bearing</td>
<td>0,006</td>
<td>0,139</td>
<td>0,002</td>
<td>0,008</td>
</tr>
<tr>
<td></td>
<td>Pitch motor</td>
<td>0,000</td>
<td>0,020</td>
<td>0,005</td>
<td>0,000</td>
</tr>
<tr>
<td></td>
<td>Pitch inverter</td>
<td>0,640</td>
<td>0,120</td>
<td>0,005</td>
<td>0,000</td>
</tr>
<tr>
<td></td>
<td>Pitch local control</td>
<td>0,680</td>
<td>0,010</td>
<td>0,005</td>
<td>0,000</td>
</tr>
<tr>
<td></td>
<td>Pitch back up power</td>
<td>0,120</td>
<td>0,100</td>
<td>0,003</td>
<td>0,000</td>
</tr>
<tr>
<td></td>
<td>Pitch communication and slip ring</td>
<td>0,100</td>
<td>0,008</td>
<td>0,000</td>
<td>0,000</td>
</tr>
<tr>
<td>Blade</td>
<td>Blade structure</td>
<td>0,000</td>
<td>0,200</td>
<td>0,045</td>
<td>0,040</td>
</tr>
<tr>
<td>Hub</td>
<td></td>
<td>0,000</td>
<td>0,185</td>
<td>0,010</td>
<td>0,005</td>
</tr>
<tr>
<td>Tower</td>
<td></td>
<td>0,000</td>
<td>0,190</td>
<td>0,010</td>
<td>0,000</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td></td>
<td><strong>8,69</strong></td>
<td><strong>3,27</strong></td>
<td><strong>0,48</strong></td>
<td><strong>0,11</strong></td>
</tr>
</tbody>
</table>

Table 8.2. Repair Categories, INNWIND.EU 10MW DTU RWT. (Hendriks, 2015)

<table>
<thead>
<tr>
<th>Category</th>
<th>Name</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Manual Restart</td>
<td>Requires physical presence of maintenance staff but not actual repair. Delay in restarting the wind turbine due to the time needed to the fault/alarm identification.</td>
</tr>
<tr>
<td>2</td>
<td>Minor Repairs</td>
<td>Faults typically involving single components or parts manageable by the wind turbine winch (typically &lt;300 kg).</td>
</tr>
<tr>
<td>3</td>
<td>Major Repairs</td>
<td>Major part repaired or replaced such as gearbox, converter or generator. Jack-up vessel is usually not required.</td>
</tr>
<tr>
<td>4</td>
<td>Major Replacement</td>
<td>Heavy operations typically requiring jack-up vessel. No Jack-up for MRS.</td>
</tr>
</tbody>
</table>
• Production based Availability is calculated and used as a measure for comparison:

\[
\text{Availability} = \frac{\text{Produced Power}}{\text{Power Production Potential}}
\]

Assumptions for MRS simulation:
1. Failure rates and repair categories are given below. Vessel types, repair durations and vessel weather limits are taken from (Dinwoodie, et al., 2015), see Table 8.3.

**Table 8.3. 20MW MRS O&M considerations**

<table>
<thead>
<tr>
<th></th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
<th>Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repair Time</td>
<td>3 hours</td>
<td>8 hours</td>
<td>26 hours</td>
<td>52 hours</td>
</tr>
<tr>
<td>Req. technicians</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Vessel Type</td>
<td>CTV</td>
<td>CTV</td>
<td>FSV</td>
<td>FSV</td>
</tr>
<tr>
<td>Weather Limits</td>
<td>Hs=1.5 m</td>
<td>Hs=1.5 m</td>
<td>Hs=1.5 m</td>
<td>Hs=1.5 m</td>
</tr>
<tr>
<td>Mob. Time</td>
<td>0</td>
<td>0</td>
<td>3 weeks</td>
<td>3 weeks</td>
</tr>
<tr>
<td>Failure Rate / turbine / year</td>
<td>8.69</td>
<td>3.27</td>
<td>0.48</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Total** 12.55

2. Failure Rates indicated above are the annul failure rate of individual 444kW machines. The total annual failure rate for the MRS structure therefore theoretically would become 45x12.55=564.75. The failure rate per unit of small 444kW wind turbines is expected to be lower than that of the big 20MW single rotor machine (Jamieson, et al., 2014), therefore the reduction of small wind turbine failure rates will be investigated.

3. No particular failure rate is assigned to the lattice support structure.

4. A “Minimum capacity for Repair” is selected as 90% - when 0.9x45~=-5 rotors of the MRS have failed a search for weather windows is initiated, the MRS continues operating with 5 rotors stopped until a Crew Transfer/Field Support Vessel arrives at the turbine.

5. If additional rotors fail while waiting for weather window, they are repaired during the upcoming repair trip. The number of required technicians is increased accordingly.

6. All 45 rotors are shut down for the duration of repair activities for safety (Figure 8.1).
As an example, Figure 8.1 shows a 6 rotor MRS where “Minimum capacity at Repair” is set to >=0.5. Therefore, until rotors 1-2 (rows 2 and 3 in Figure 8.1) fail at hours ~50 and ~100, no repair action is initiated but when a third rotor fails (row 1 in Figure 8.1) at ~200 hours, a search for weather window is initiated. When the technicians arrive at the site of the MRS, turbine it is stopped for the repair duration (short “0” condition in rows 4-6 in Figure 8.1).

![Image](image.png)

**Figure 8.1. Example of 6 rotor system MRS (1 – operating, 0 – not operating).**

No jack-up vessel is used for major replacements. It is assumed that there is a crane integrated in the MRS capable of handling individual 444kW machines. (Jamieson, et al., 2014). A scaled down 10MW reference power curve is used for each small rotor to calculate total MRS power production (Figure 8.2), (Jamieson, et al., 2014).
Figure 8.2. 10MW DTU RWT and 444kW single MRS rotor power curves.

Assumptions for MRS simulation:
1. Failure rates and repair categories are given below, adopted from (Dinwoodie, et al., 2015):

Table 8.4. 2x10MW DTU RWT O&M considerations.

<table>
<thead>
<tr>
<th></th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
<th>Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Man.</td>
<td>Restart</td>
<td>Minor rep.</td>
<td>Major rep.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Repair Time</td>
<td>3 hours</td>
<td>8 hours</td>
<td>26 hours</td>
<td>52 hours</td>
</tr>
<tr>
<td>Req. technicians</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Vessel Type</td>
<td>CTV</td>
<td>CTV</td>
<td>FSV</td>
<td>HLV</td>
</tr>
<tr>
<td>Weather Limits</td>
<td>Hs=1.5 m</td>
<td>Hs=1.5 m</td>
<td>Hs=1.5 m</td>
<td>Hs=2 m Ws=10m/s</td>
</tr>
<tr>
<td>Mob. Time</td>
<td>0</td>
<td>0</td>
<td>3 weeks</td>
<td>2 months</td>
</tr>
<tr>
<td>Failure Rate / turbine / year</td>
<td>8.69</td>
<td>3.27</td>
<td>0.48</td>
<td>0.11</td>
</tr>
</tbody>
</table>

2. Two 10MW turbines are used to represent a 20MW turbine, both at the same point and hub height (119 m). Turbine specifications are taken from (Bak, et al., 2013).
3. Jack-up vessel (HLV) is used for Category 4 replacement.

8.3. Results

The effect on the system availability when the number of rotors in MRS is increased was investigated and the results are given below, in Figure 8.3. The availability is calculated throughout the 25 years of turbine lifetime, minimum MRS Capacity at Repair was set to 1, meaning that every failure was repaired immediately without
grouping (after every individual 444kW rotor failure a weather window search was initiated).

Figure 8.3. Effect of increasing the number of rotors in MRS.

It can be observed that with increasing number of rotors in the MRS the overall availability is decreasing. This is due to the fact that with every failure of one rotor, the whole 20MW MRS system has to be shut down for the duration of repair activities (safety). When the number of rotors is reduced, the availability obviously tends to that of 2x10MW DTU reference turbine because the initial failure rates for individual machines are identical (MRS and DTU reference), see Table 8.3 and Table 8.4.

Multiple simulations were run in order to determine how the total power output of the MRS changes when repairs of individual rotors are grouped together. Minimum MRS Capacity at Repair range was set from 0.4 to 1, which corresponds to requirement of 27 to 1 individual rotors to fail before repair activities are planned and weather windows search is initiated.

In terms of availability, it is clearly visible in Figure 8.4 that MRS is not performing as well as 2x10MW DTU Reference turbines. This is mainly due to the fact, as discussed above, that all the MRS rotors have to be stopped when repairs are conducted on failed rotors in order to ensure safe working environment.
Figure 8.4. MRS availability, 2x10MW DTU reference for comparison.

It should be mentioned that despite the fact that the MRS availability is ~5% lower than that of 2x10MW DTU reference turbines, there is no need for heavy lifting vessels for Category 4 repairs. This would reduce the repair costs significantly (to prove this further investigations including costs of vessel lease and labour is needed).

Since the MRS system can function with some of individual 444kW rotors in failure (shut down) it is possible to find the optimal point where the number of trips to the MRS and 2x10MW DTU Reference turbines is the same. At this point the costs of repairs are expected to be in favour of the MRS system again due to the fact that heavy lift vessels are not necessary.
In Figure 8.5 the optimum point is found to be at 0.7 minimum MRS Capacity at Repair, this corresponds to waiting for 13 individual 444kW rotors to fail before initiating repair activities. Although, it has to be noted that at 0.7 Minimum MRS Capacity at Repair almost 10% of availability is lost. At this point it is not clear whether the optimum point in Figure 8.5 is the most cost optimal, but it can be clearly stated that moving “1” from minimum MRS Capacity at Repair to “0.9” would reduce the number of trips to the turbine by almost 3 times and would significantly reduce the repair expenditure. In order to find the most cost optimal approach to optimum minimal MRS Capacity at Repair, a LCOE analysis has to be performed.

As was mentioned in the previous chapter and in (Jamieson, et al., 2014), the reliability of individual small 444kW rotors is expected to increase (due to smaller scale, mass production and quality control) and thus failure rates would be lower than those of large 10/20MW wind turbines. The following Figure 8.6 shows the effect of failure rate reduction in terms of Production Based Availability. The model was set to run for 25 years of turbine lifetime with minimum MRS Capacity at Repair set to 0.9 (minimum 5 failed rotors before weather windows search is initiated).
It can be concluded that with increase in small rotor reliability the availability is expected to rise, although not linearly. This is due to the fact that that downtime after a failure is mainly driven by the waiting time rather than the repair time (see Figure 8.7).
It is also evident from the Figure 8.7 that with increasing number of rotors allowed to fail before repair activities are initiated on the MRS (decreasing minimum MRS Capacity at Repair) the contribution of waiting time becomes more important in the total downtime. This is easily explained by the fact that there are less trips to the turbine site.

There is generally no way of reducing the waiting time for weather windows, other than designing vessels that can withstand harsher sea/weather conditions and, possibly, having more precise weather forecasts. Although, keeping in mind that repair duration contributes to ~25% (at 0.8-1 minimum MRS Capacity at Repair) of total downtime of the MRS there is room for improvement. It would be possible to further reduce the downtime due to repair time by implementing a smart procedure of repair instead of complete a shutdown of the MRS for repair duration, for example (proposed by Peter Jamieson):

1. Shutting down the MRS and locking the yawing mechanism.
2. Under automatic control and using the overhead travelling crane, release the nacelle mountings and electrical connection of a faulty MRS turbine and drop that rotor nacelle assembly (RNA) to base.
3. Resume operation of the MRS array while doing repairs to the faulty system or preparing a replacement RNA.
4. When ready, stop array and reversing the procedure 1, install repaired/replacement RNA.
5. Finish or lower next failed RNA to repeat procedure.

This procedure would be very beneficial in Category 2 and 4 repairs, which have a long duration (26-52 hours), although Category 1 (manual restarts) could be completed without lowering the individual RNAs because the restart and inspection in Category 1 only takes 3 hours. When it comes to Category 2 repairs, this procedure would be beneficial if the dismounting, drop down/lift-up and remounting of the RNAs would take relatively small amount of time compared to the 8 hour assumed repair time.

It also should be mentioned that the direct decrease in overall availability of the MRS (in comparison with 10MW DTU reference) does not necessarily translate in immediate losses of power production. The MRS is capable of operating at partial capacity with multiple failed individual rotors if the weather conditions are favorable for power production. Based on weather forecast, repairs should be scheduled for in periods when the weather conditions are not good for power production.
APPENDIX A. RELIABILITY ASSESSMENT OF STRUCTURAL COMPONENTS

A.1. Reliability Analysis of Structural Components with Non-Linear Failure Functions - FORM

In general failure functions for structural components are non-linear and the safety margin $M = g(X)$ is thus not normally distributed.

A first approximation to obtain an estimate of the reliability index in this case could be to linearize the safety margin with the point corresponding to the expected values as expansion point:

$$M \cong g(\mu_X) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i} \bigg|_{X_i=\mu_i} (X_i - \mu_i)$$  \hspace{1cm} (A.1)

The reliability index can then be estimated by assessing linear safety margins. However, as noted above, the failure surface $g(x) = 0$ can be defined by many different but equivalent failure functions.

This implies that the reliability index based on the linearized safety margin becomes dependent on the mathematical formulation of the safety margin. This problem is also known as the invariance problem.

In (Hasofer & Lind, 1974) proposed a definition of the reliability index which is invariant with respect to the mathematical formulation of the safety margin.

First, it is assumed that the stochastic variables $X_i, i = 1, \ldots, n$ are statistically independent. Further, it is implicitly assumed that the variables are normally distributed. The first step in calculation of the so-called Hasofer & Lind reliability index $\beta_{HL}$ is to define a transformation from $X$ to stochastic variables $U$ that are normalized. The normalized variables $U_i, i = 1, \ldots, n$ with expected values 0 and standard deviation 1 are defined by:

$$U_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \hspace{1cm} i = 1, 2, \ldots, n$$  \hspace{1cm} (A.2)

By this transformation the failure surface in the new u-space is given by, see Figure A.1:

$$g(\mu_{X_i} + \sigma_{X_i} u_1, \ldots, \mu_{X_n} + \sigma_{X_n} u_n) = g_u(u) = 0$$  \hspace{1cm} (A.3)
It should be noted that the $u$-space is rotationally symmetric with respect to the standard deviations.

The Hasofer & Lind reliability index $\beta$ is defined as the smallest distance from the origin $O$ in the $u$-space to the failure surface $g_u(u) = 0$. This is illustrated in Figure A.2. The point $A$ on the failure surface closest to the origin is denoted the $\beta$-point or the design point. The Hasofer & Lind reliability index defined in the $u$-space is invariant to different equivalent formulations of the failure function because the definition of the reliability index is related to the failure surface and not directly to the failure function. The reliability index is thus defined by the optimization problem:

$$\beta = \min_{g_u(u) = 0} \sqrt{\sum_{i=1}^{n} u_i^2}$$  \hspace{1cm} (A.4)

The solution point for $u$ is denoted $u^*$, see figure 3.6.

**Figure A.9.** Geometrical illustration of the reliability index $\beta$. 

**Figure A.8.** Failure functions in the $x$-space and the $u$-space.
The numerical calculation of the reliability index $\beta$ defined by (A.4) can be performed in a number of ways. (A.4) is an optimization problem with a quadratic objective function and one non-linear constraint. A number of algorithms exist for solution of this type of problem, e.g. the NLPQL algorithm by (Schittkowski, 1985). Here a simple iterative algorithm will be described. For simplicity the index $u$ will be omitted on the failure function $g(u)$ in the following.

At the $\beta$ point $u^*$ it is seen that the following relation must be fulfilled:

$$ u^* = \lambda \nabla g(u^*) $$

(A.5)

where $\lambda$ is a proportionality factor. In order to formulate an iteration scheme it is assumed that a point $u^0$ close to $u^*$ is known, i.e.:

$$ u^* = u^0 + \Delta u $$

(A.6)

A first order approximation of $g(u)$ in $u^0$ then gives:

$$ g(u^*) \approx g(u^0) + \nabla g(u^0)^T (u^* - u^0) = g(u^0) + \nabla g(u^0)^T \Delta u $$

(A.7)

Application of (3.22) and (3.23) gives:

$$ g(u^*) \approx g(u^0) + \nabla g(u^0)^T (u^* - u^0) \approx g(u^0) + \nabla g(u^0)^T (\lambda \nabla g(u^0) - u^0) $$

(A.8)

from which $\lambda$ can be determined using that $g(u^*) = 0$:

$$ \lambda = \frac{\nabla g(u^0)^T u^0 - g(u^0)}{\nabla g(u^0)^T \nabla g(u^0)} $$

(A.9)

The following iteration scheme can then be formulated

1. Guess ($u^0$)
   Set $i = 0$

2. Calculate $g(u^i)$

3. Calculate $\nabla g(u^i)$

4. Calculate an improved guess of the $\beta$ point using (A.22) and (A.23)

$$ u^{i+1} = \nabla g(u^i) \frac{\nabla g(u^i)^T u^i - g(u^i)}{\nabla g(u^i)^T \nabla g(u^i)} $$

(A.10)

5. Calculate the corresponding reliability index

$$ \beta^{i+1} = \sqrt{(u^{i+1})^T u^{i+1}} $$

(A.11)

6. If convergence in $\beta$ (e.g. if $|\beta^{i+1} - \beta^i| \leq 10^{-3}$), then stop, else $i = i + 1$ and go to 2.

If a unit normal vector $a$ to the failure surface at the $\beta$ point $u^*$ is defined by:
\[ \mathbf{a} = -\frac{\nabla g(\mathbf{u}^*)}{|\nabla g(\mathbf{u}^*)|} \] \hspace{1cm} (A.12)

then the \( \beta \)-point \( \mathbf{u}^* \) can be written, see (A.5):

\[ \mathbf{u}^* = \beta \mathbf{a} \] \hspace{1cm} (A.13)

It is noted that \( \mathbf{a} \) is directed towards the failure set. The safety margin corresponding to the tangent hyperplane obtained by linearizing the failure function at the \( \beta \) point can then be written:

\[ M = \beta - \mathbf{a}^T \mathbf{U} \] \hspace{1cm} (A.14)

Further, using that \( \mathbf{a}^T \mathbf{a} = 1 \) it is seen from (A.13) that the reliability index \( \beta \) can be written:

\[ \beta = \mathbf{a}^T \mathbf{u}^* \] \hspace{1cm} (A.15)

For fixed \( \mathbf{a} \) it is seen that:

\[ \frac{d\beta}{du_i} \bigg|_{u_i-u_i^0} = \alpha_i \] \hspace{1cm} (A.16)

i.e. the components in the \( \mathbf{a} \) vector can be considered measures of the relative importance of the uncertainty in the corresponding stochastic variable on the reliability index. However, it should be noted that for dependent (correlated) basic variables the components in the \( \mathbf{a} \)-vector cannot be linked to a specific basic variable, see the next section.

An important sensitivity measure related to \( \alpha_i \) is the so-called omission sensitivity factor \( \varsigma_i \) suggested by (Madsen, 1988). This factor gives the relative importance on the reliability index by assuming that stochastic variable no. \( i \), i.e. it is considered a deterministic quantity. If variable no. \( i \) is applied to the value \( u_i^0 \), then the safety margin in the normalized space is written:

\[ M_i' = \beta - \alpha_i u_i^0 - \sum_{j=1}^{n} \alpha_j U_j \] \hspace{1cm} (A.17)

with the reliability index:

\[ \beta_i' = \frac{\beta - \alpha_i u_i^0}{\sqrt{1 - \alpha_i^2}} \] \hspace{1cm} (A.18)

The omission sensitivity factor \( \varsigma_i \) is defined by:
\[ \zeta_i = \frac{\beta' - \alpha_u u_i^0 / \beta}{\sqrt{1 - \alpha_i^2}} \]  
(A.19)

If especially \( u_i^0 = 0 \) is chosen, then:

\[ \zeta_i = \frac{1}{\sqrt{1 - \alpha_i^2}} \]  
(A.20)

It is seen that if \( |\alpha_i| < 0.14 \), then \( \zeta_i - 1 < 0.01 \), i.e. the error in the reliability index is less than 1% if a variable with \( |\alpha| < 0.14 \) is fixed. The omission sensitivity factor can be generalized to non-normal and dependent stochastic variables, see (Madsen, 1988).

In this section it is assumed that the stochastic variables are normally distributed. The normalized variables \( U \) defined by the linear transformation (A.2) are thus also normally distributed. If the failure function in the \( u \)-space is not too non-linear, then the probability of failure \( P_f \) can be estimated from:

\[ P_f = P(M \leq 0) \simeq P(\beta - \alpha^T U \leq 0) = \Phi(-\beta) \]  
(A.21)

where \( \Phi \) is the standard normal distribution function.

**A.2. Reliability Index for Correlated, Normally Distributed Variables**

Let the stochastic variables \( X_i, \ i = 1, \ldots, n \) be normally distributed with expected values \( \mu_{X_i}, \ldots, \mu_{X_n} \), standard deviations \( \sigma_{X_1}, \ldots, \sigma_{X_n} \) and with correlation coefficients \( \rho_{ij}, i, j = 1, \ldots, n \). Further, let a failure function \( g(x) \) be given. In order to determine a reliability index for this failure mode a transformation from correlated to uncorrelated stochastic variables is added to the procedure described in previous section. This transformation can be performed in several ways, e.g. by determining eigenvalues and eigenvectors, see (Thoft-Christensen & Baker, 1982). Here Choleski triangulation is used. The procedure described in the following requires that the correlation coefficient matrix \( \rho \) is positive definite.

The first step is to determine normalized variables \( Y_i, i = 1, \ldots, n \) with expected value 0 and standard deviation 1:

\[ Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad , \quad i = 1, \ldots, n \]  
(A.22)
It is easy to see that $Y$ will have a covariance matrix (and correlation coefficient matrix) equal to $\rho$.

The next step is to define a transformation from $Y$ to uncorrelated and normalized variables $U$ with expected values 0 and standard deviations 1. The transformation is written:

$$Y = TU$$  \hspace{1cm} \text{(A.23)}

where $T$ is a lower triangular matrix (i.e. $T_{ij} = 0$ for $j > i$). It is seen that the covariance matrix $C_Y$ for $Y$ can be written:

$$C_Y = E[YY^T] = E[TTU^TUU^T] = TT^T = \rho$$  \hspace{1cm} \text{(A.24)}

The elements in $T$ are then determined from $TT^T = \rho$ as:

$$T_{11} = 1$$

$$T_{21} = \rho_{12} \quad T_{22} = \sqrt{1 - T_{21}^2}$$

$$T_{31} = \rho_{13} \quad T_{32} = \frac{\rho_{23} - T_{21}T_{31}}{T_{22}} \quad T_{33} = \sqrt{1 - T_{31}^2 - T_{32}^2}$$  \hspace{1cm} \text{(A.25)}

etc.

The transformation form $X$ to $U$ can now be written:

$$X = \mu_X + DTU$$  \hspace{1cm} \text{(A.26)}

where $D$ is a diagonal matrix with standard deviations in the diagonal. Using the failure function can be written $g(x) = g(\mu_X + DTU)$ and a reliability index $\beta$ can be determined as shown in the above section.

**A.3. Reliability Index for Independent, Non-Normally Distributed Variables**

Generally the stochastic variables are not normally distributed. In order to determine a measure of the reliability of a component (failure mode) with non-normally distributed variables it is natural, as for normally distributed variables, to establish a transformation to standardized (uncorrelated and normalized) normally distributed variables and to determine a Hasofer & Lind reliability index $\beta$. 
A simple transformation from $X_i$ to $U_i$ can be defined by:

$$
\Phi(U_i) = F_{X_i}(X_i)
$$

(A.27)

where $F_{X_i}$ is the distribution function for $X_i$. Given a realization $u$ of $U$ a realization $x$ of $X$ can be determined by:

$$
x_i = F_{X_i}^{-1}(\Phi(u_i))
\vdots
x_n = F_{X_n}^{-1}(\Phi(u_n))
$$

(A.28)

and the failure surface can be written:

$$
g(x_1,\ldots,x_n) = g(F_{X_1}^{-1}(\Phi(u_1)),\ldots,F_{X_n}^{-1}(\Phi(u_n))) = 0
$$

(A.29)

In the algorithm for determination of $\beta$ (see section A.1) the gradient of the failure function with respect to $u_i$ is needed. From (A.46):

$$
\frac{\partial g}{\partial u_i} = \frac{\partial g}{\partial x_i} \frac{\partial x_i}{\partial u_i} = \frac{\partial g}{\partial x_i} \frac{\Phi^{-1}(F_{X_i}(x_i))}{f_{X_i}(x_i)}
$$

(A.30)

where $f_{X_i}(x_i) = dF_{X_i}(x_i)/dx_i$ is the density function for $X_i$.

**A.4. Reliability Index for Dependent, Non-Normally Distributed Variables**

In this section two techniques are described, which can be used to determine a reliability index when the stochastic variables are dependent and non-normally distributed, namely methods based on the Rosenblatt transformation, see (Rosenblatt, 1952) and the Nataf transformation, see (Nataf, 1962).

For dependent stochastic variables $X_i, i = 1,\ldots,n$ the Rosenblatt transformation, see (Rosenblatt, 1952) , can be used to define a transformation to the $u$-space of uncorrelated and normalized normally distributed variables $U_i, i = 1,\ldots,n$. The transformation is defined as:
\[ x_1 = F^{-1}_{X_i}(\Phi(u_1)) \]
\[ x_2 = F^{-1}_{X_2|X_1}(\Phi(u_2) | X_1 = x_1) \]
\[ \vdots \]
\[ x_n = F^{-1}_{X_n|X_{i-1},\ldots,X_n}(\Phi(u_n) | X_1 = x_1,\ldots,X_{n-1} = x_{n-1}) \]  

(A.31)

where \( F_{X_i|X_1,\ldots,X_{i-1}}(x_i | X_1 = x_1,\ldots,X_{i-1} = x_{i-1}) \) is the distribution function of \( X_i \) given \( X_1 = x_1,\ldots,X_{i-1} = x_{i-1} \):

\[ F_{X_i|X_1,\ldots,X_{i-1}}(x_i | X_1 = x_1,\ldots,X_{i-1} = x_{i-1}) = \frac{\int_{-\infty}^{x_i} f_{X_i|X_1,\ldots,X_{i-1}}(x_1,\ldots,x_{i-1},t) dt}{f_{X_i,\ldots,X_n}(x_1,\ldots,x_{i-1})} \]  

(A.32)

\( f_{X_1,\ldots,X_i}(x_1,\ldots,x_i) \) is the joint density function of \( X_1,\ldots,X_i \). The transformation starts for given \( u_1,\ldots,u_n \) by determination of \( x_1 \). Next \( x_2 \) is calculated using the value of \( x_1 \) determined in the first step. \( x_3,\ldots,x_n \) are then calculated in the same stepwise manner.

The inverse transformation from \( x_1,\ldots,x_n \) to \( u_1,\ldots,u_n \) is defined by:

\[ u_1 = \Phi^{-1}(F_{X_1}(x_1)) \]
\[ u_2 = \Phi^{-1}(F_{X_2|X_1}(x_2 | X_1 = x_1)) \]
\[ \vdots \]
\[ u_n = \Phi^{-1}(F_{X_n|X_{i-1},\ldots,X_n}(x_n | X_1 = x_1,\ldots,X_{n-1} = x_{n-1})) \]  

(A.33)

The Rosenblatt transformation is very useful when the stochastic model for a failure mode is given in terms of conditional distributions. For example, this is often the case when statistic uncertainty is included.
APPENDIX B. SYSTEM MODELLING AND CALCULATION OF RELIABILITY BOUNDS FOR SYSTEMS

B.1. Modelling of Series Systems

A failure element or component can be interpreted as a model of a specific failure mode at a specific location in the structure.

Figure B.10. Failure element

The combination of failure elements in a series system can be understood from the statically determinate (non-redundant) truss-structure in Figure B.2 with n structural elements (trusses). Each of the n structural elements is assigned 2 failure elements. One with a failure function modelling material yielding failure and one with a failure function modelling buckling failure.

Figure B.11. Statically determinate truss structure.

For such a statically determinate structure it is clear that the whole structural system fails as soon as any structural element fails, i.e. the structure has no load-carrying capacity after failure of one of the structural elements. This is called a weakest link system and is modelled as a series system. The series system which then becomes the systems reliability model consists of 2n failure elements shown in Figure B.3.

Figure B.12. Weakest link system modelled as a series system of failure elements.

It is in this connection important to notice the difference between structural components and failure elements and the difference between a structural system and a systems reliability model.

If failure of one failure element is defined as systems failure the reliability of the series system can be interpreted as the reliability of failure. That also includes the case of statically indeterminate structures where failure of more than one failure element cannot be accepted.
In the following, so-called simple bounds and Ditlevsen bounds will be introduced as bounds for the reliability of series systems.

**Simple Bounds**

Simple bounds can be introduced as:

\[ \max_{i=1}^{m} P(M_i \leq 0) \leq P_f^S \leq \sum_{i=1}^{m} \left( P(M_i \leq 0) \right) \]  
(B.1)

where the lower bound corresponds to the exact value of \( P_f^S \) if all the elements in the series system are fully correlated.

In the terms of reliability indices (B.1) can be written:

\[ -\Phi^{-1} \left( \sum_{i=1}^{m} \Phi(-\beta_i) \right) \leq \beta^S \leq \min_{i=1}^{m} \beta_i \]  
(B.2)

When the failure of one failure element is not dominating in relation to the other failure elements the simple bounds are generally too wide and therefore often of minor interest for practical use.

**Ditlevsen Bounds**

Much better bounds are obtained from the second-order bounds called Ditlevsen bounds (Ditlevsen, 1979). The derivation of the Ditlevsen bounds can be seen in (Madsen, et al., 1986), (Ditlevsen, 1979), (Thoft-Christensen & Baker, 1982), (Thoft-Christensen & Murotsu, 1986) or (Ditlevsen & Madsen, 1990). The bounds are:

\[ P_f^S \geq P(M_1 \leq 0) + \sum_{i=2}^{m} \max_{i \neq j} \left\{ P(M_i \leq 0) - \sum_{j=1}^{i-1} P(M_i \leq 0 \cap M_j \leq 0) \right\} \]  
(B.3a)

\[ P_f^S \leq \sum_{i=1}^{m} P(M_i \leq 0) - \sum_{i=2}^{m} \max_{i \neq j} \left\{ P(M_i \leq 0 \cap M_j \leq 0) \right\} \]  
(B.3b)

and in terms of the FORM approximation in reliability indices:

\[ \Phi(-\beta^S) \geq \Phi(-\beta_1) + \sum_{i=2}^{m} \max_{i \neq j} \left\{ \Phi(-\beta_i) - \sum_{j=1}^{i-1} \Phi_2(-\beta_i, -\beta_j; \rho_{ij}) \right\} \]  
(B.4a)

\[ \Phi(-\beta^S) \leq \sum_{i=1}^{m} \Phi(-\beta_i) - \sum_{i=2}^{m} \max_{i \neq j} \left\{ \Phi_2(-\beta_i, -\beta_j; \rho_{ij}) \right\} \]  
(B.4b)

The numbering of the failure elements influences the bounds. However, experience suggests that it is a good choice to arrange the failure elements according to decreasing probability of failure, i.e. \( P(M_1 \leq 0) \geq P(M_2 \leq 0) \geq \cdots \geq P(M_m \leq 0) \). The Ditlevsen bounds are usually much more precise than the simple bounds in (B.1)-(B.2), but
require the estimation of the two-dimensional distribution function $\Phi_{2}(\beta_i, \beta_j; \rho_{ij})$ in (B.4).

**B.2. Modelling of Parallel Systems**

The introduction and the necessity of parallel systems for the reliability modelling of some structural systems can be illustrated by considering the statically indeterminate (redundant) truss-structure in Figure B.4 with $N$ structural elements (trusses). Two failure elements are assigned to each of the $N$ structural elements, one with a failure function modelling material yielding failure and one with a failure function modelling buckling failure.

![Figure B.13. Statically indeterminate truss structure.](image)

For such a statically indeterminate (redundant) structure it is clear that the whole structural system will not always fail as soon as one of structural element fails, because the structure has a load-carrying capacity after failure of some of the structural elements. This load-carrying capacity is obtained after a redistribution of the load effects in the structure after the element failure. Failure of the entire redundant structure will then often require failure of more than one structural element. (It is in this connection very important to define exactly what is understood by failure of the structural system). Clearly the number of systems failure modes in a redundant structure is generally high. Each of these system failure modes can be modelled by a parallel system consisting of generally $n$ elements, where $n$ is the number of failure elements which have to fail in the specific systems failure mode before the entire structure is defined to be in a state of failure. The parallel system with $n$ elements is shown in Figure B.5.

![Figure B.14. Failure mode of a redundant structure modelled as a parallel system.](image)

Since a redistribution of the load effects has to take place in a redundant structural system after failure of one or more of the structural elements it becomes very important in parallel systems to describe the behavior of the failed structural elements after failure.
has taken place. If the structural element has no strength after failure the element is said to be perfectly brittle. If the element after failure has a load-bearing capacity equal to the load at failure, the element is said to be perfectly ductile.

In Figure B.6 a perfectly brittle and a perfectly ductile element are shown with an example of the behaviors and the symbols used for perfectly brittle and perfectly ductile elements, respectively.

![Perfectly brittle and perfectly ductile elements with symbols.](image)

In the following, simple bounds and a second order bound will be introduced as bounds for the reliability of parallel systems.

**Simple Bounds**

If only the active constraints assumed to influence the reliability of the parallel system the simple bounds can be introduced as:

\[
0 \leq P_f^P \leq \min_{i=1}^{n_A} \left( P(M_i^f \leq 0) \right)
\]  

(B.11)

where \( M_i^f \), \( i = 1, \ldots, n_A \) are the linearized safety margins at the joint \( \beta \)-point. The upper bound corresponds to the exact value of \( P_f^P \) if all the \( n_A \) elements are fully correlated with \( \rho_{ij} = 1 \).

In the terms of reliability indices \( \beta^f \) (B.10) can be written:

\[
\max_{i=1}^{n_A} \beta_i^f \leq \beta^P \leq \infty
\]

(B.12)

If all correlation coefficients \( \rho_{ij} \) between the \( n_A \) elements are higher than zero, the following simple bounds are obtained:

\[
\prod_{i=1}^{n_A} P(M_i^f \leq 0) \leq P_f^P \leq \min_{i=1}^{n_A} P(M_i^f \leq 0)
\]

(B.13)
where the lower bound corresponds to uncorrelated elements, i.e., \( \rho_{ij} = 0 \), \( i \neq j \). In terms of \( \beta^j \), (B.12) becomes:

\[
\max_{i=1}^{n_j} \beta^j_i \leq \beta^P \leq -\Phi^{-1}\left(\prod_{i=1}^{n_j} \Phi(-\beta^j_i)\right)
\]

(B.14)

The simple bounds will in most cases be so wide that they are of little practical use.

**Second-Order Upper Bound**

A second-order upper bound of \( P_f^P \) can be derived as:

\[
P_f^P \leq \min_{i,j=1}^{n_j} P(M^j_i \leq 0 \cap M^j_j \leq 0)
\]

(B.15)

The corresponding lower bound of \( \beta^P \) is:

\[
\beta^P \geq -\Phi^{-1}\left(\max_{i,j=1}^{n_j} \Phi_2(-\beta^j_i, -\beta^j_j, \rho_{ij})\right)
\]

(B.16)

In (B.16) it is seen that the probability of failure of a parallel system of two elements \( \Phi_2(-\beta^j_i, -\beta^j_j, \rho_{ij}) \) is necessary. These probabilities are the same as the probabilities used in the Ditlevsen bounds for series systems. Hereby the tools for evaluation of the bounds are described.

More refined and complicated bounds can also be developed, see (Thoft-Christensen & Murotsu, 1986), but will not be shown here.

**B.3. Advanced Asymptotic Methods**

It has already been mentioned that the bounds methods can be used in hand calculations. However, in professional reliability programs (e.g., SYSREL, STRUREL and COMREL) other more precise and more refined methods are used. Two of these methods are the Hohenbichler approximation, see (Hohenbichler, 1984), and the approximation (Gollwitzer & Rackwitz, 1986). These methods are in general very precise and make it possible to calculate \( \Phi_m \) within reasonable computer time.
APPENDIX C. LIFE CYCLE MODELLING

C.1. Introduction

In this appendix it is described how life cycle modelling can be performed at 4 different levels of complexity.

C.2. Crude deterministic formulation

In a crude deterministic formulation generic models for the costs are formulated directly as function of the design parameters and using basic up-scaling laws adjusted for technology improvement effects. The optimal design is obtained as the design which minimizes the cost of energy expressed as the total expected costs per MWh (levelised production costs, \( \text{LPC} \)) with benefits and costs obtained during the lifetime capitalized to year 0 (time of decision):

\[
\begin{align*}
\text{min}_{z} \quad \text{LPC}(z) &= \text{min}_{z} \frac{C_f(z, X_d) - \sum_{t=0}^{T_f} B_t(z, X_d)(1 + r)^{-t}}{\sum_{t=0}^{T_f} B_t(z, X_d)(1 + r)^{-t}}
\end{align*}
\]

where

- \( C_f \) is the total discounted costs during the design lifetime \( T_f \) (not incl. cost of evt. collapse)
- \( B \) is the total expected energy production during the design lifetime
- \( r \) is the real rate of interest, i.e. adjusted for inflation
- \( C_{OM,t} \) is the initial (manufacturing and installation) costs – corresponds to CAPEX
- \( C_{OM,t} \) is the costs for operation and maintenance in year \( t \) \( \sum_{t=0}^{T_f} C_{OM,t}(z, X_d)(1 + r)^{-t} \).

Demolition costs are assumed to be included in \( C_{OM,T_f} \). \( C_{OM,t} \) corresponds to OPEX
- \( B_t \) is the energy production in year \( t \)

It is noted that the \( \text{LPC} \) is closely connected to the Cost of Energy (\( \text{COE} \)) defined by

\[
\text{COE}(z) = \frac{\text{CAPEX} + \text{OPEX}}{\sum_{t=0}^{T_f} B_t(z, X_d)} = \frac{C_f + \sum_{t=0}^{T_f} C_{OM,t}(z, X_d)(1 + r)^{-t}}{\sum_{t=0}^{T_f} B_t(z, X_d)}
\]

with the only difference that the energy production is not capitalized. Further, the optimization problem in (C.1) is closely related to the general cost-benefit optimization problem. The only difference is that instead of optimizing the difference between costs and benefits, the ratio between them in (LPC) in (C.1) is optimized, and evt. collapse costs are not included in (C.2).
C.3. Deterministic, design based on requirements in standards

In the deterministic, code / standard-based formulation uncertainties are taken into account through partial safety factors and appropriate characteristic values of the stochastic variables. The optimal design is obtained as the design which minimize \( LPC \) (levelised production costs) taking into account design requirements specified in standards, e.g. IEC 61400-1. The design requirements could e.g. be related to maximum allowable design stresses and strains in the tower, blades, etc. The optimization problem can be formulated as:

\[
\min \ z \quad \frac{C_T(z, X_d)}{B(z, X_d)}
\]

subject to \( G_{F,i}(z, X_d) \geq 0 \), \( i = 1, \ldots N_D \)

where

\( z \) = design values of stochastic variables \( X \) obtained using characteristic values (e.g. 5% quantiles for strength parameters) and partial safety factors \( \gamma \) for load and strength parameters

\( G_{F,i}(z, X_d) \) = design equation for component \( i \), e.g. check of buckling or fatigue

C.4. Reliability based design

In a reliability-based formulation uncertainties are modelled by stochastic models and reliability of the wind turbine components are directly included. The optimal design is obtained as the design which minimizes \( LPC \) (levelised production costs) during the lifetime taking into account minimum requirements to the reliability (though maximum acceptable probabilities of failure):

\[
\min \ z \quad \frac{C_T(z, X_d)}{B(z, X_d)}
\]

subject to \( P_{F,i}(z) \leq P_{\text{max}, F.i} \), \( i = 1, \ldots N_R \)

where

\( C_T(z, X_d) \) = the total discounted costs during the design lifetime \( T_L \) (not incl. cost of evt. collapse)

\( P_{F,i}(z) \) = probability of failure of component \( i \)

\( P_{\text{max}, F.i} \) = maximum acceptable failure rate of component \( i \)

Note that design values of the stochastic variables are used in the objective function. Instead of a formulation based on requirements to component reliabilities, a formulation using a systems reliability requirement can be used.
The maximum acceptable probability of failure, $P_{max,F}$ should take into account consequences to economic losses and eventual risk of loss of human lives. Typically, in case of risk of human lives $P_{max,F,i} = 10^{-5}$ per year and if no risk of human lives $P_{max,F,i} = 10^{-4}$ per year (relative cost of safety measure: normal). For wind turbines the risk of loss of human lives in case of failure of structural components is often negligible.

C.5. Risk based design

In the risk based formulation uncertainties are also modelled by stochastic models and reliability of the wind turbine components are directly included. The optimal design is obtained as the design which minimizes the expected value of LPC taking into account eventual minimum (human related) safety requirements to the reliability:

$$\min_z \quad E[LPC(z, X)] = E \left[ \frac{C_T(z, X)}{B(z, X)} \right]$$

subject to $P_{F,i}(z) \leq P_{max,F,i}, \ i = 1, \ldots, N_R$

where

- $E[\cdot]$ expectation with respect to the stochastic variables $X$
- $B(z, X)$ benefits / income from energy production discounted to time of decision
- $C_T(z, X)$ is the total discounted costs during the design lifetime $T_L$ (not incl. cost of evt. collapse)
- $P_{F,i}(z)$ probability of failure of component $i$
- $P_{max,F,i}$ maximum acceptable failure rate of component $i$

Instead of a formulation based on requirements to component reliabilities, a formulation using a systems reliability requirement can be used.

The maximum acceptable probability of failure, $P_{max,F,i}$ should take into account the risk of eventual loss of human lives. Typically, in case of risk of human lives $P_{max,F,i} = 10^{-5}$ per year and if no risk of human lives, see section 3.3.
APPENDIX D. RELIABILITY UPDATING

D.1. Introduction

When new information it can be used to update the stochastic models and the estimates of the reliability (probability of failure). In this note it is described how this updating can be performed when the new information consists of

1. Observation of events described by one or more stochastic variables. The observation is modeled by an event margin and the failure event by a safety margin. Updated / conditional probabilities of failure can then be obtained, see section D.2.
2. Samples / measurements of a stochastic variable $X$. Updating can in this case be performed using Bayesian statistics.

D.2. Bayesian updating of stochastic variables

In order to model the observed events an event function

$$H = h(X)$$ (D.1)

is introduced. The event function $h$ corresponds to the limit state function. The actual observations are considered as realizations (samples) of the stochastic variable $H$. This type of information can e.g. be

- Inspection events such as measurements of the chloride content in concrete structures or measurements of crack sizes in steel structures exposed to fatigue loading. The event margin can include the uncertainty related to the measurement.
- Proof loading where a well-defined load is applied to a structure and the level of damage (usually no damage is observed) is observed.
- Repair events where a certain type of repair or maintenance has been performed.
- No-failure events where the ‘simple’ observation that the structure / component considered is well-functioning after some time in use.

It is assumed that these observations can be modeled by
a. inequality events $\{H \leq 0\}$, i.e. it is observed that the observed quantity is less than or equal to some limit, or
b. equality events $\{H = 0\}$, i.e. it is observed that the observed quantity is equal to some limit.

If inequality events are used the updated probability of failure is estimated by

$$P_F^U = P(g(X) \leq 0 \mid h(X) \leq 0) = \frac{P(g(X) \leq 0 \cap h(X) \leq 0)}{P(h(X) \leq 0)}$$ (D.2)
where \( M = g(X) \) is the safety margin related to the limit state function \( g(x) \) and \( X = (X_1, \ldots, X_n) \) are stochastic variables. In (D.2) it is used that the probability of an event \( A \) given an event \( B \) (denoted \( P(A|B) \)) is equal to \( \frac{P(A \cap B)}{P(B)} \). It is seen that \( P(g(X) \leq 0 \cap h(X) \leq 0) \) is the probability of a parallel system with two elements. (D.2) can be evaluated by simulation or FORM/SORM methods, see (Madsen, 1987).

Other observations can be modeled by equality events \( \{H = 0\} \), i.e. it is observed that the observed quantity is equal to some limit. In this case the updated probability of failure can be estimated by, see (Madsen, et al., 1986), (Madsen, 1987) and (Schall & Rackwitz, 1988).

\[
P_{f_{f}} = P(g(X) \leq 0 \mid h(X) = 0) = \frac{P(g(X) \leq 0 \cap h(X) = 0)}{P(h(X) = 0)} = \left. \frac{\partial}{\partial z} P(g(X) \leq 0 \cap h(X) \leq z) \right|_{z=0} \frac{\partial}{\partial z} P(h(X) \leq z) \right|_{z=0} \tag{D.3}
\]

Equation (D.3) can also be evaluated by FORM/SORM methods and can easily be generalized if more than one event is observed. In most software packages for reliability analysis efficient algorithms are available for solving this problem.

If possible Bayesian techniques should be used for parameter estimation because Bayesian estimation gives an estimate of the statistical uncertainty related to the estimated parameters and because updating of the model when new information becomes available is easy.

If observations of one (or more) of the stochastic variables \( X \) are available, the probabilistic model can be updated and thereby also the probability of failure. Consider a stochastic variable \( X \) with density function \( f_{x}(x) \). If \( q \) denotes a vector of parameters defining the distribution for \( X \), the density function of the stochastic variable \( X \) can be written

\[
f_{x}(x, q) \tag{D.4}
\]

If \( X \) is normally distributed then \( q \) could contain the mean and the standard deviation of \( X \).

If the parameters \( q \) are uncertain then \( f_{x}(x, q) \) can be considered as a conditional density function : \( f_{x}(x|Q) \) and \( q \) denotes a realization of \( Q \). The initial density function for the parameters \( Q \) is denoted \( f'_{Q}(q) \) and is denoted the \textbf{prior density function}. 

(INNWIND.EU, Deliverable 1.34, Integrated system reliability analysis)
It is assumed that \( n \) observations (realizations) of the stochastic variable \( X \) are available making up a sample \( \hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) \). The realizations are assumed to be independent. The updated density function \( f_Q^*(q|\hat{x}) \) of the uncertain parameters \( Q \) given the realizations is denoted the posterior density function and is given by, see textbook on Bayesian statistics, e.g. (Box & Tiao, 1992) and (Lindley, 1976).

\[
f_Q^*(q|\hat{x}) = \frac{f_N(\hat{x}|q)f_Q^*(q)}{\int f_N(\hat{x}|q)f_Q^*(q) dq}
\]  

(D.5)

where \( f_N(\hat{x}|q) = \prod_{i=1}^{N} f_X(\hat{x}_i|q) \) is the probability density at the given observations assuming that the distribution parameters are \( q \). The integration in (12.5) is over all possible values of \( q \).

The updated density function of the stochastic variable \( X \) given the realization \( \hat{x} \) is denoted the predictive density function and is defined by,

\[
f_X(x|\hat{x}) = \int f_X(x|q)f_Q^*(q|\hat{x}) dq
\]  

(D.6)

Given the distribution function for the stochastic variable \( X \), the prior distribution is often chosen such that the posterior distribution will be of the same type as the prior distribution (a so-called conjugated prior). In the literature a number of prior, posterior and predictive distribution functions can be found, see e.g. (Raiffa & Schlaifer, 1968), (Aitchison & Dunsmore, 1975) and (Rackwitz & Schrupp, 1985).

By use of the Bayesian method presented here, both the physical uncertainty related to the considered variable as well as the statistical uncertainty related to the model parameters can be quantified. However, as mentioned the probabilistic model must also be formulated such that the measurement uncertainty and the model uncertainty are taken into account.

Due to the ability of Bayesian statistics to incorporate engineering judgement and experience into the statistical modeling, different reassessment engineers may reach different results due to the use of different statistical models. This may obviously be a serious obstacle for the use of such methods. Also in order to avoid this obstacle it is necessary to define and agree on a common basis for such analyses thus ensuring that reliability based reassessment analyses are performed on a consistent and comparable basis.
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